

# Evolutionary Computation Applied to the Tuning of MEMS Gyroscopes

Didier Keymeulen, Wolfgang Fink, Michael I. Ferguson, Chris Peay, Boris Oks, Richard Terrile, and Karl Yee

Jet Propulsion Laboratory  
California Institute of Technology  
Pasadena, CA 91109, USA  
+1 (818) 354-4280  
didier.keymeulen@jpl.nasa.gov

## ABSTRACT<sup>1</sup>

We propose a tuning method for MEMS gyroscopes based on evolutionary computation to efficiently increase the sensitivity of MEMS gyroscopes through tuning and, furthermore, to find the optimally tuned configuration for this state of increased sensitivity. The tuning method was tested for the second generation JPL/Boeing Post-resonator MEMS gyroscope using the measurement of the frequency response of the MEMS device in open-loop operation.

**Categories & Subject Descriptors:** B.6.3 Verilog

**General Terms:** Algorithms

## Keywords

Genetic Algorithms, Simulated Annealing, Dynamic Hill Climbing, Evolvable Hardware, Gyroscope, MEMS.

## 1. INTRODUCTION

Future NASA missions would benefit tremendously from an inexpensive, navigation grade, miniaturized inertial measurement unit (IMU), which surpasses the current state-of-the art in performance, compactness and power efficiency. Towards this end, under current development at JPL's MEMS Technology Group are several different designs for high performance, small mass and volume, low power MEMS gyroscopes. The accuracy with which the rate of rotation of micro-gyros can be determined depends crucially on the properties of the resonant structure. It is expensive to attempt to achieve these desired characteristics in the fabrication process, especially in the case of small MEMS structures, and thus one has limited overall sensor performance.

The sensitivity of the MEMS gyroscope is maximized when the resonant frequencies of the two modes of freedom of a MEMS gyroscope are identical. Symmetry of construction is necessary to attain this degeneracy. However, despite a symmetric design, perfect degeneracy is never attained in practice. Tuning of the gyros into the state of degeneracy is achieved through application of bias voltages on built-in tuning pads to electrostatically soften the mechanical

springs. Because of the time consuming nature of the tuning process when performed manually, in practice any set of bias voltages that produce degeneracy is viewed as acceptable at the present time. A need exists for reducing the time necessary for performing the tuning operation, and for finding the optimally tuned configuration, which employs the minimal maximum tuning voltage.

This paper describes the application of evolutionary computation to this optimization problems. Evolutionary computation was first introduced by John Holland in USA [5] and Rechenberg [11] and Schwefel [12] in Europe to solve optimization problems. Our method used as a fitness function for each set of bias voltages applied to built-in tuning pads, the evaluation of the frequency split between the two modes of resonance of the MEMS gyroscope. Our evaluation proceeds in two steps. First, it measures the frequency response using a dynamic signal analyzer. Second, it evaluates the frequency of resonance of both modes by fitting Lorentzian curves to the experimental data. The process of setting the bias voltages and the evaluation of the frequency split is completely computer automated. The computer controls a signal analyzer and power supplies through General Purpose Interface Bus (GPIB). Our method has demonstrated that we can obtain a frequency split of 52mHz in one hour compared with 200mHz obtained manually by humans in several hours.

## 2. TEST SETUP FOR GYRO TUNING

Two stochastic optimization methods based on evolutionary computation, namely dynamic hill climbing and simulated annealing algorithms, have been implemented on a dedicated, JPL/Boeing-specific hardware platform, which is described in the following sections.

### 2.1 Mechanism of the JPL MEMS Micro-gyro

The mechanical design of the JPL MEMS microgyro can be seen in Figure 1. The JPL/Boeing MEMS post resonator gyroscope (PRG), is a MEMS analogue to the classical Foucault pendulum. A pyrex post, anodically bonded to a silicon plate, is driven into a rocking mode along an axis (labeled as X in Figure 1) by sinusoidal actuation via electrodes beneath the plate. In a rotating reference frame the post is coupled to the Coriolis force, which exerts a tangential "force" on the post. Another set of electrodes beneath the device senses this component of motion along an axis (labeled as Y in the figure) perpendicular to the driven motion. The voltage that is required to null out this motion is directly proportional to the rate of rotation to which the device is subjected. A change in capacitance occurs as the top plate vibrates due to the oscillating gap variation

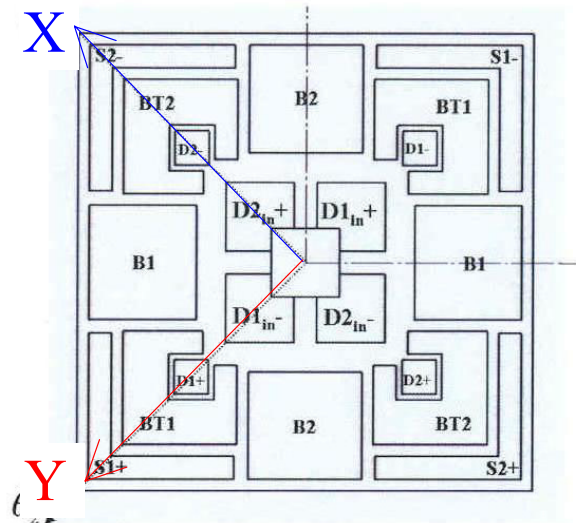
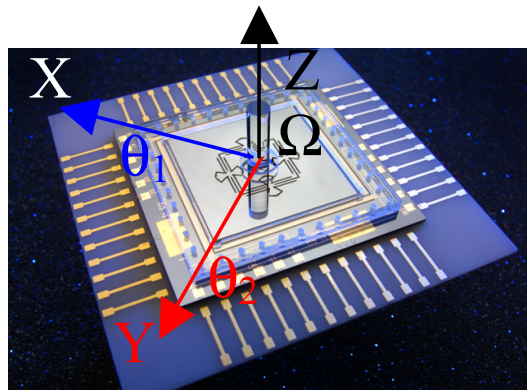
---

Copyright 2005 Association for Computing Machinery. ACM acknowledges that this contribution was authored or co-authored by a contractor or affiliate of the U.S. Government. As such, the Government retains a nonexclusive, royalty-free right to publish or reproduce this article, or to allow others to do so, for Government purposes only.

GECCO'05, June 25–29, 2005, Washington, DC, USA.

Copyright 2005 ACM 1-59593-010-8/05/0006...\$5.00.

between this plate and the electrodes underneath. This change in capacitance generates a time-varying sinusoidal charge that can be converted to a voltage using the relationship  $V=Q/C$ . The post can be driven around the drive axis by applying a time-varying voltage signal to the drive petal electrodes labeled D1-, D1+, D1in- and D1in+ in Figure 1. Because there is symmetry in the device, either of the two axes can be designated as the drive axis. Each axis has a capacitive petal for sensing oscillations as well; driving axis: labeled S1+ and S1- in Figure 1, sensing axis: labeled S2+ and S2- in Figure 1. The micro-gyro has additional plates that allow for electro-static softening of the silicon springs, labeled B1, BT1, B2, and BT2 in Figure 1. Static bias voltages can be used to modify the amount of softening for each oscillation mode. In an ideal, symmetric device, the resonant frequencies of both modes are equal; however, unavoidable manufacturing imperfections in the machining of the device can cause asymmetries in the silicon structure of the devices, resulting in a *frequency split* between the resonant frequencies of these two modes.



**Figure 1.** A magnified picture of the JPL MEMS micro-gyroscope with sense axis Y (S2-, S2+ electrodes used to sense, D2-, D2+, D2in- and D2in+ used to drive along the sense axis) and drive axis X (D1-, D1+, D1in- and D1in+ used to drive, S1-, S1+ electrodes used to sense along the drive axis) and the electrodes used for biasing (B1, B2, BT1, BT2) (picture courtesy of C. Peay, JPL).

The frequency split reduces the mechanical coupling between the two modal axes, and thus reduces the sensitivity for detection of rotation. By adjusting the static bias voltages on the capacitor plates, frequencies of resonance for both modes are modified to match each other; this is referred to as the tuning of the device.

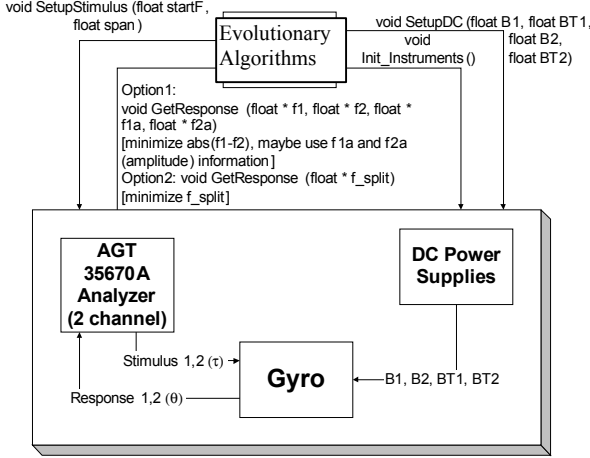
In order to extract the resonant frequencies of the vibration modes, there are two general methods: 1) open-loop and 2) closed-loop control [9]. In an open-loop system, we are measuring the frequency response along the drive axis over a 50Hz band and extract from the measurement the frequency split. A faster method is a closed-loop control, whereby the gyro is given an impulse disturbance and is allowed to oscillate freely between the two resonance frequencies, using a hardware platform, controlling the switch of the drive-angles [10].

## 2.2 Instrumentation Platform

The measurement consists of exciting the drive axis with a sine wave at a given frequency and measuring the resulting amplitude. This is done repeatedly through the frequency spectrum. Because of cross-coupling between the different axes, two peaks in the amplitude response will appear at two different frequencies, showing the resonant frequencies of both axes (Figure 7). This takes approximately 1.4 minutes to complete using our instrumentation platform and must be done at least three times to average out noise.

The platform includes one GPIB programmable power supply DC voltage, a GPIB signal analyzer to extract frequency responses (from 3.3kHz to 3.35kHz) of the gyro in open-loop, and a computer (PC) to control the instruments and execute the optimization algorithms. The power supply DC voltage controls the static bias voltages (connected to the plate B1, BT1, B2, and BT2 in Figure 1) that are used to modify the amount of damping to each oscillation mode. The GPIB signal analyzer generates a sine wave with a variable frequency (from 3300 Hz to 3350 Hz and a step of 62.5 mHz – 800 points, 50Hz span) on the drive electrode (D1-, D1+, D1in- and D1in+ in Figure 1) and measures the response signal on the sense electrode (S1-, S1+ in Figure 1) along the drive axis X.

A PC runs the instrument control tool, the measurement tool, and the evolutionary computation tool. The instrument control software sets up the static bias voltages using the GPIB power supply DC voltage and measures the frequency response along the X axis using the GPIB signal analyzer as shown in Figure 2. The measurement tool software calculates the frequency split using peak fitting algorithms. Finally the evolutionary computation tool software determines the new DC bias voltages from the frequency split. This procedure is repeated until a satisfactory frequency split is obtained.



**Figure 2. Software interface between the Simulated Annealing/Modified Genetic Algorithm and the Instrumentation Platform using a GPIB programmable power supply DC voltage and a signal analyzer. The Simulated Annealing and GA/Hill Climbing are running on a PC, which controls the bias voltages and receives the frequencies of both resonance modes.**

### 3. OVERVIEW OF ALGORITHMS

As with most prototype sensor development efforts, the JPL/Boeing MEMS post-resonator gyroscope is custom-built and hand-tuned for optimal performance. Many methods have been developed for tuning MEMS post-resonator gyroscopes. For example [1] and [2] use adaptive and closed-loop methods while [3] changes the frame of the pick-off signal. As explained in Section 2, our approach of gyro tuning consists of adjusting 4 bias voltages between -60V and +14V on capacitor plates to reduce the frequency split. Our approach can be seen as an optimization problem where the value to minimize is the difference between two frequencies and the parameters are the four bias voltages. We propose two *stochastic optimization algorithms* – *dynamic hill climbing* and *simulated annealing* used for different space applications [4], to optimize the values of bias voltages applied to built-in tuning pads on the MEMS post-resonator gyroscope.

#### 3.1 Dynamic Hill Climbing Algorithm

To reduce the number of evaluations of potential solutions we used a Dynamic Hill Climbing introduced by Yuret et al. [6]. We start with a potential solution represented by a point in a four dimensional search space that is easiest to implement (from the hardware standpoint). Our initial point is obtained by setting the static bias voltages (connected to the plate B1, BT1, B2, and BT2 in Figure 1) to [+14Volt] [+14Volt] [+14Volt] [+14Volt]. It is therefore setting the potential between the bias plates and the common plate to zero since the common plate is held +14 volts. We then use a hill climbing method to find the best neighboring point. In our four dimensional space we examine eight neighboring points (+/- step size in each dimension, one dimension at a time). Original value of step size is 10 volts. Once an optimum point is found with the original step size (no neighboring points give a higher fitness), the step size is halved and the hill climbing is restarted using the half step size. The step size is continually halved until it is equal to 0.1 volts, we accept the optimum at this point and the location of this optimum is saved. To avoid multiple evaluations of the same points

which are time expensive, the results of all evaluations are cached. If the found optimum is of an un-acceptable fitness, a new starting point is found by computing the location that is farthest in the search space from all known local optima. The step size is set at a value of 10 volts and we repeat the process until a desired optimum is found.

#### 3.2 Simulated Annealing Algorithms

Simulated annealing (SA) is a widely used and well-established optimization technique especially for high-dimensional configuration spaces [7, 8]. The goal is to minimize an energy function  $E$ , akin to the fitness function used in GAs, which is a function of  $N$  variables, with  $N$  being usually a large number. In our case, the energy function is the frequency split of the MEMS micro-gyroscope, and  $N=4$  is the number of the bias voltages. The minimization is performed by randomly changing the value of one of the 4 bias voltages and reevaluating the energy function  $E$ , i.e., the resulting frequency split. Two cases can occur: 1) the change in the bias voltage results in a new, smaller frequency split; or 2) the resulting frequency split is larger or unchanged. In the first scenario the new set of bias voltages is stored and the change accepted. In the second scenario, the new set of bias voltages is only stored with a certain likelihood (Boltzmann probability, including an annealing temperature). This ensures that the overall optimization algorithm does not “get stuck” in local minima too easily, as is the case with greedy downhill optimization, and hence to fail to find the desired global minimum. The annealing temperature directly influences the Boltzmann probability by making it less likely to accept an energetically unfavorable step, the longer the optimization lasts (cooling schedule). Then the overall procedure is repeated until the annealing temperature has reached its end value, or a preset number of iterations has been exceeded, or the energy function  $E$ , i.e., the frequency split, has reached an acceptable level. In contrast to standard simulated annealing, we apply a modified simulated annealing related algorithm, in which the annealing temperature oscillates between to boundaries (reheating / recooling), with the lower boundary being a fixed, small value  $> 0$  and the upper boundary being the current best frequency split. The exit criterion in our case is the reaching of a desired minimal frequency split.

### 4. EVALUATION OF THE FREQUENCY SPLIT

We have developed a method to extract the peak from the frequency response using a peak fitting algorithm based on a model of the transfer function of the MEMS micro-gyro [9]. The transfer function  $H(s)$  for a single input- single output (SISO) system, modeling the gyro behavior, is described in Equation 1. The transfer function relates the angle velocity of the drive axes  $\dot{\theta}_1$  and the torque in the drive axes  $\tau_1$ . The constants in the equation are as follows:

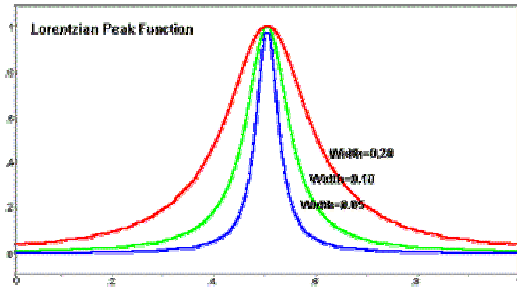
- $\omega_d$  : natural resonance of the sensor axis’ driving mode
- $Q_d$  : quality factor depends on the spring constant
- $J_t$  : symmetric moments of inertia around the X and Y axes

$$H(S) = \frac{\left(\frac{1}{J_t}\right)S}{S^2 + \left(\frac{\omega_d}{Q_d}\right)S + \omega_d^2} \quad (1)$$

The SISO model is approximated by the Lorentzian function shown in Equation 2 and illustrated in Figure 3. It is characterized by having a very sharp and narrow peak with most of the intensity of the peak located in the tails (or “wings”), extending to infinity (Figure 3).

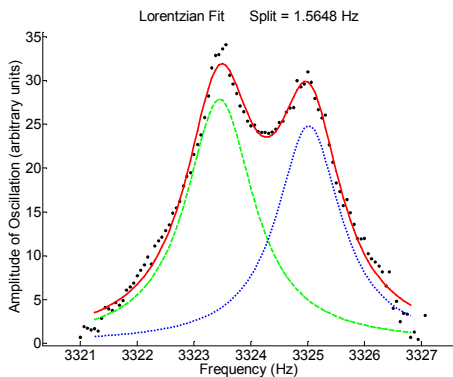
$$f(x) = \frac{H}{4\left(\frac{x-X_0}{W}\right)^2+1} \quad (2)$$

where  $X_0$  = Peak Position,  $H$  = Peak Height,  $W$  = Full width at Half Height.



**Figure 3. Lorentzian functions with different values used to approximate the transfer function of a SISO system, modeling the MEMS micro-gyro behavior.**

The objective is to fit the sum of two Lorentzian functions to the frequency response experimental data as illustrated in Figure 4. We can then calculate the frequency split between the two modes of resonance by measuring the difference of the peak locations of each Lorentzian function.



**Figure 4. Fitting the sum of two Lorentzian functions to the frequency response experimental data. The green and blue curves are Lorentzian functions and the red curve is the sum of the two functions. The experimental data points are plotted in black.**

Each Lorentzian function is completely described by 3 parameters as shown in Equation 2. To come up with two Lorentzian functions, the sum of which best describes the experimental data, we have to optimize a total of 6 parameters.

Lorentzian 1: location ( $L1$ ), width ( $W1$ ), height ( $H1$ )

Lorentzian 2: location ( $L2$ ), width ( $W2$ ), height ( $H2$ )

We have used the Chi Square statistic to measure the agreement between *actual/experimental/measured* data and

*calculated/Lorentzian* model. The Chi Squared is calculated as follows:

$$\chi^2 = \frac{\sum_{i=0}^n \left( \frac{Actual_i - Calculated_i}{RMSNoise} \right)^2}{n-f} \quad (3)$$

where  $RMSNoise$  is the estimated Root Mean Squared noise in the *actual* data. The variable  $n$  is the number of data points and  $f$  is the total number of variables in the Lorentzian model, i.e., 6.

The process of optimizing the 6 parameters in the Lorentzian model is described below:

Start at default values for all 6 variables.

Fixed an initial value for the step size

Loop until step size < threshold

1. Vary  $L1$  and  $L2$  by one step size while keeping all other parameters at their best values, calculate the Lorentzian model and evaluate the  $\chi^2$  of the fittest curves
  - a. Save values of  $L1$  and  $L2$  that produce lowest  $\chi^2$
2. Vary  $H1$  and  $H2$  one step size while using best parameters obtained so far, calculate the Lorentzian model and evaluate the  $\chi^2$  of the fittest curves.
  - a. Save values of  $H1$  and  $H2$  that produce lowest  $\chi^2$
3. Vary  $W1$  and  $W2$  one step size while using best parameters obtained so far, calculate the Lorentzian model and evaluate the  $\chi^2$  of the fittest curves.
  - a. Save values of  $W1$  and  $W2$  that produce lowest  $\chi^2$

Save the set of variable  $L1$ ,  $L2$ ,  $H1$ ,  $H2$ ,  $W1$ ,  $W2$  giving the best  $\chi^2$ .

If the variation of  $\chi^2$  from previous iteration is equal to zero then decrease step size

Once all six parameters are determined, the frequency split is found by taking the difference between  $L1$  and  $L2$ .

## 5. RESULTS OF EVOLUTIONARY COMPUTATION

The MEMS post resonator micro-gyroscope, developed by the MEMS Technology Group at JPL, is subject to an electro-static fine-tuning procedure, performed by hand, which is necessary due to unavoidable manufacturing inaccuracies. In order to fine-tune the gyro, 4 voltages applied to 8 capacitor plates have to be determined within a range of  $-60V$  to  $+15V$ , respectively. The manual hand tuning took several hours and obtained a frequency split of 200 mHz.

In order to fully automate the time-taking manual fine-tuning process, we have established a hardware/software interface to the existing manual gyro-tuning hardware-setup using commercial-off-the-shelf (COTS) components described in Section 4.

We developed and implemented two stochastic optimization techniques for efficiently determining the optimal tuning voltages and incorporated them in the hardware/software interface: a modified *simulated annealing related algorithm* and a modified *genetic algorithm with limited evaluation*.

## 5.1 Simulated Annealing Approach

We were able to successfully fine-tune both MEMS post-resonator gyroscope and MEMS disk-resonating gyroscope (a different gyro-design not discussed here) within one hour for the first time fully automatically. After only 49 iterations with the modified Simulated Annealing related optimization algorithm we obtained a frequency split of 125mHz within a 1V-discretization of the search space, starting with an initial split of 2.625Hz, using a 50Hz span and 800 points on the signal analyzer for the MEMS post-resonator gyroscope (Figure 5 top). For the MEMS disk-resonating-gyroscope we obtained a frequency split of 250mHz/500mHz within a 0.1V-/0.01V-discretization of the search space, starting with an initial split of 16.125Hz/16.25Hz, after 249/12 iterations using a 200Hz span and 800 points on the signal analyzer (Figure 5 bottom). All three results are better than what can be accomplished manually but worse than the results obtained by dynamic hill climbing. The results can be further improved, using the peak fitting algorithm employed in the dynamic hill climbing approach.

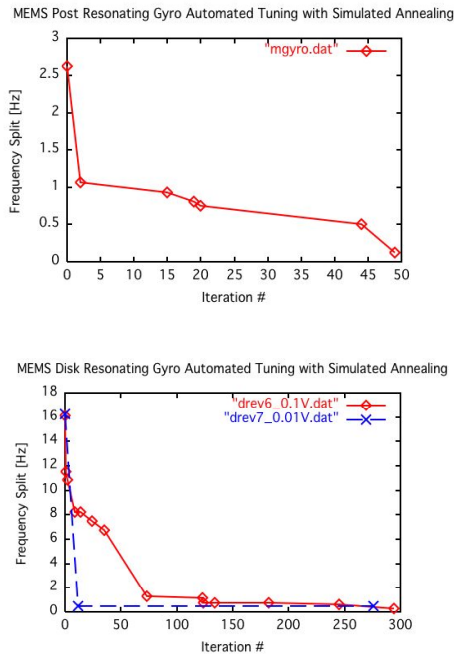


Figure 5. Frequency split as a function of Simulated Annealing Iterations: (top) for the MEMS post-resonator gyroscope; (bottom) for the MEMS disk-resonating gyroscope.

## 5.2 Modified Genetic Algorithm Approach

We were also able to fine-tune the MEMS post-resonator gyroscope within one hour fully automatically using a genetic related algorithm: dynamic hill climbing. Figure 6 shows the progress of the optimization algorithm aimed at minimizing the frequency split. Each evaluation is a proposed set of bias voltages. Our optimization method only needed 47 evaluations (51 min) to arrive at a set of bias voltages that produced a frequency split of less than 100mHz.

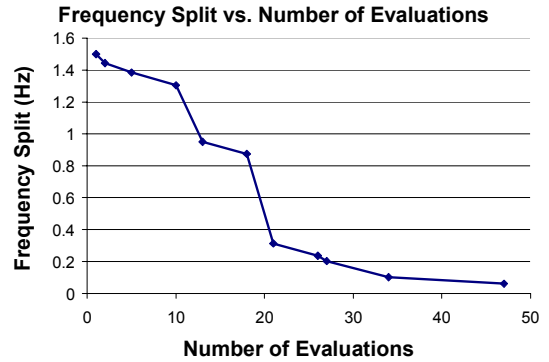


Figure 6. Frequency split as a function of number of evaluations used by the Dynamic Hill Climbing algorithm.

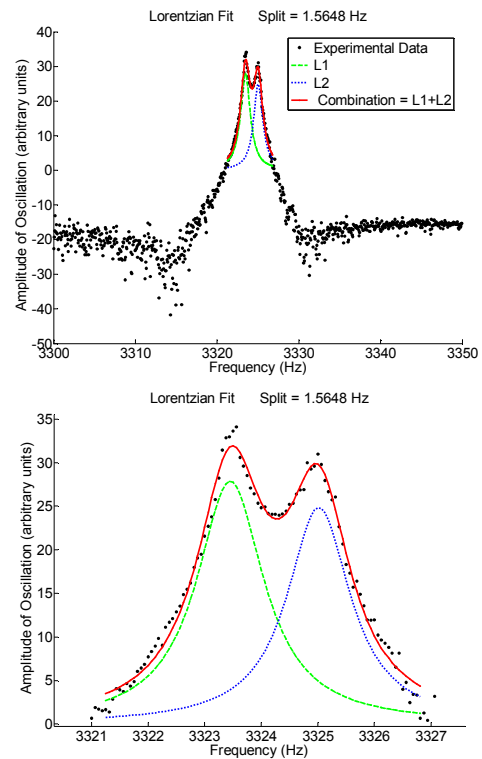
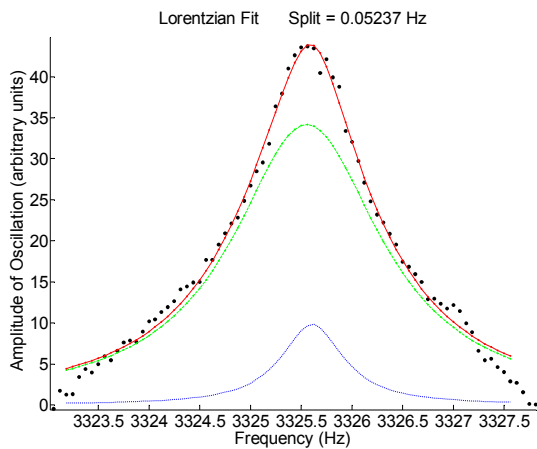
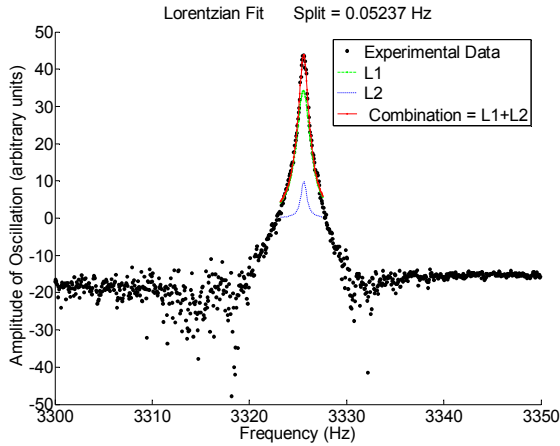


Figure 7. Frequency response (top: 50Hz band, bottom: 6Hz band) before tuning using the modified genetic algorithm. The frequency split is 1555mHz. The Y axis is measured in dB. The initial values of the four bias voltages are: B1 = 14.00V BT1 = 14.00V B2 = 14.00V BT2 = 14.00V. The bottom picture shows a zoom of the frequency split over a 6Hz band.

Figures 7 and 8 show the frequency response for the unbiased micro-gyro respectively before and after tuning using the dynamic hill climbing and the peak fitting algorithm.

After optimization of the bias voltages (Figure 8), the frequency split has been minimized to less than 100mHz and the two peaks are indistinguishable on an HP spectrum analyzer at 62.5mHz / division (50Hz span, 800 points) setting, which was used during the optimization process.



**Figure 8. Frequency response (top: 50Hz band, bottom: 5Hz band) after tuning using the modified genetic algorithm. The Y axis is measured in dB. The tuning frequency split is 52mHz. The optimized values of the four bias voltages are: B1 = 4.00V BT1 = 4.00V B2 = 14.00V BT2 = -16.00V. The bottom picture shows a zoom of the frequency split over a 4Hz band.**

The frequency split of 52mHz was verified using a higher resolution of the signal analyzer.

## 6. CONCLUSION

The tuning method for MEMS micro-gyroscopes based on evolutionary computation shows great promise as a technology to replace the cumbersome human manual tuning. We demonstrate that we can, for the first time fully automatically, obtain a four times smaller frequency split at a tenth of the time, compared to human performance. The novel capability of fully automated gyro tuning enables ultra-low mass and ultra-low-power high-precision Inertial Measurement Unit (IMU) systems to calibrate themselves autonomously during ongoing missions, e.g., Mars Ascent Vehicle. Our current effort is producing an even faster tuning algorithm based on a closed-loop measurement, which can be implemented on a single FPGA chip next to the gyro [10].

## 7. ACKNOWLEDGMENTS

The work described in this publication was carried out at the Jet Propulsion Laboratory, California Institute of Technology, under a contract with the National Aeronautics and Space Administration. Special thanks to Tom Prince, who has supported this research through the Research and Technology Development grant entitled “Evolutionary Computation Technologies for Space Systems” and to Thomas George, head of the MEMS Technology Group at JPL, who has encouraged the research from its beginning.

## 8. REFERENCES

- [1] Leland, R.P., “Adaptive mode tuning vibrational gyroscopes”, IEEE Trans. Control Systems Tech., vol. 11, no. 2, pp242-247, March 2003.
- [2] Painer C.C., Shkel A.M., “Active structural error suppression in MEMS vibratory rate integrating gyroscopes”, IEEE Sensors Journal, vol.3, no.5, pp. 595-606, Oct. 2003.
- [3] Y. Chen, R. M’Closkey, T. Tran and B. Blaes. “A control and signal processing integrated circuit for the JPL-Boeing micromachined gyroscopes” (submitted to IEEE)
- [4] R. J. Terrile, et al., “Evolutionary Computation Technologies for Space Systems”, in Proceedings of the IEEE Aerospace Conference, Big Sky, March 2005
- [5] J.H. Holland, *Adaptation in Natural and Artificial Systems*, The University of Michigan Press, Ann Arbor, Michigan, 1975.
- [6] D. Yuret, M. de la Maza, “Dynamic Hill Climbing – Overcoming limitations of optimization techniques”, AI Laboratory, MIT, Cambridge, MA 02139, USA
- [7] N. Metropolis, A.W. Rosenbluth, M.N. Rosenbluth, A.H. Teller, E. Teller, “Equation of State Calculation by Fast Computing Machines,” *J. of Chem. Phys.*, **21**, 1087--1091, 1953.
- [8] S. Kirkpatrick, C.D. Gelat, M.P. Vecchi,, “Optimization by Simulated Annealing,” *Science*, **220**, 671--680, 1983.
- [9] K. Hayworth, “Continuous Tuning and Calibration of Vibratory Gyroscopes”, In NASA Tech Brief, Oct 2003 (NPO-30449)
- [10] M.I. Ferguson et al.. “A Hardware Platform for Tuning of MEMS Devices Using Closed-Loop Frequency Response”, IEEE Aerospace 2005, Big Sky, March 2005.
- [11] I. Rechenberg, *Evolutionstrategie: Optimierung Technischer Systeme nach Prinzipien der Biologischen Evolution*, Frommann- Holzboog, Stuttgart, 1973.
- [12] H.-P. Schwefel, ‘Numerical Optimization of Computer Models’, Wiley, Chichester, 1981.