FTO: A genetic algorithm for tunnel design optimisation

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ABSTRACT

This paper presents a genetic algorithm for automating the design process for underground tunnels. The tunnel cross-sectional profile must enclose a prescribed minimum space polygon, and for stability the bending moments in the shotcrete lining must not exceed certain limits. The engineer seeks to achieve these aims while minimizing the volume of rock to be excavated. We describe the five parameters defining the tunnel profile, and develop a fitness function to model these design objectives. The genetic algorithm package, FTO, includes 2D finite element mesh generation and analysis modules, and sophisticated graphics using Microsoft .NET technology. A realistic example is solved in 30 generations, and the result compared with engineering experience.

Categories and Subject Descriptors

G.1.6-Constrained optimization; G.1.8-Finite element methods; J.2-Engineering

General Terms

Algorithms, Design, Languages

Keywords

Genetic algorithms, tunnel design, finite element method.

1. INTRODUCTION

Evolutionary algorithms have been applied to engineering optimization problems since the late 1980's. The overwhelming majority of these applications have been in electronic engineering (e.g. circuit design) and mechanical/structural engineering, however; relatively little use has been made in civil engineering, although this is now changing. Pichler et al [1] recently developed a soft-computing-based parameter identification method for the determination of material properties of the surrounding rock mass during a tunnel construction project. A neural network was used to back-analyze the rock properties from the tunnel wall displacements measured during construction. The learning weights of the NN were optimized by a genetic algorithm.

The present paper describes another, more direct application of genetic algorithms in the field of geotechnical engineering, namely to optimize the cross-sectional profile of an underground tunnel at the design stage.

Genetic and Evolutionary Computation Conference (GECCO) '05, June 25-29, 2005, Washington, DC, USA.

2. TUNNEL PROFILE

The major constraint when designing a road or rail transport tunnel, is a set of minimum width/height requirements to accommodate the traffic. A typical situation is shown in Figure 1, where the prescribed points $P_1P_2P_3P_4P_5$ define a minimum polygon which the tunnel must enclose (Assuming symmetry about the centerline P_1P_5 we deal with only the right-hand half-plane). In this graph the coordinate origin is at O. However, the centre of the tunnel can be vertically offset up or down from this point by a distance z; the tunnel shown is centred at O′.

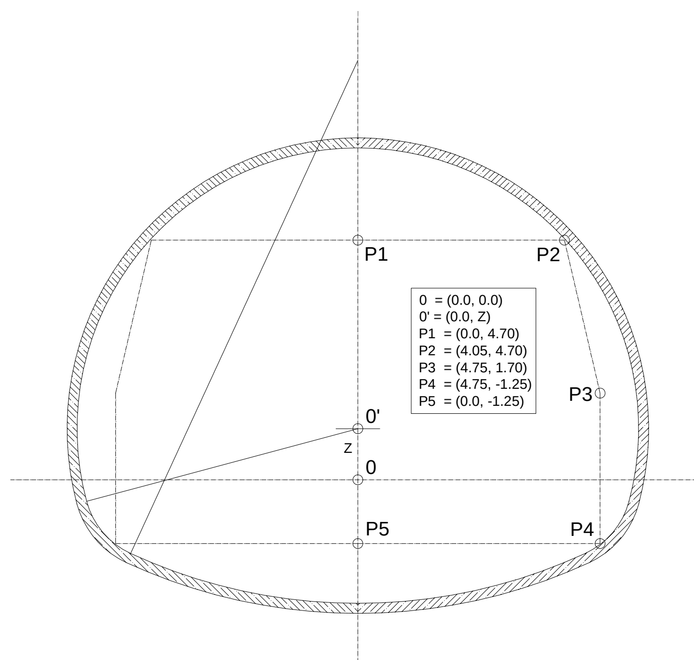

Figure 1: Tunnel profile with enclosed minimum polygon

Once the tunnel centre-point O′ is chosen, the profile is constructed as shown in Figure 2; it consists of upper, corner and lower arcs. The upper profile is an arc AB of radius r_1, centre O′; this extends to an angle α below the horizontal. On the radius OB a distance r_2 back from B is measured, giving the point E. The corner arc BC is from a circle centre E, radius r_2, subtended by an angle β. Finally, we project CE back to the centerline at F, and construct the lower arc CD centred at F, subtended by the angle γ. This profile algorithm is the standard cross-section design employed in Austria in conjunction with the New Austrian Tunnelling Method [2].

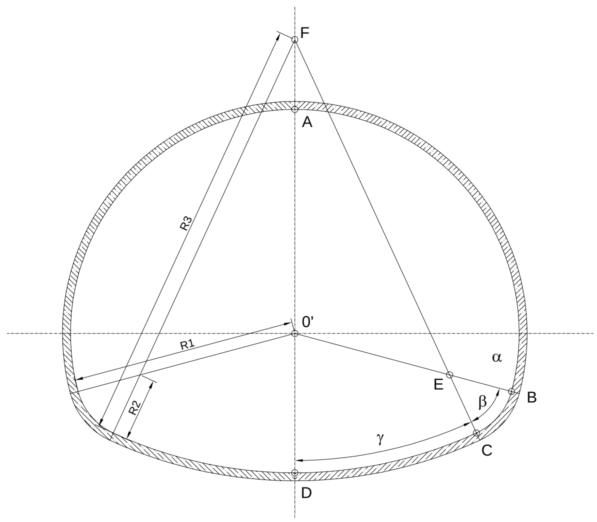

Figure 2: Geometric construction of tunnel profile

An individual tunnel is thus defined by the independent variables $\{r_1, r_{21}, \alpha, \beta, z\}$, where $r_{21} = r_2 / r_1$. The constraints on these variables are:

$$r_1 > 0 \ , \ 0 < r_{21} < 1 \ , \ -90° < \alpha < 90° \ , \ \alpha + \beta \leq 90°.$$

The third constraint allows the corner arc BEC to lie above the centre level, if $\alpha < 0$.

The optimal tunnel profile depends on the horizontal and vertical *in situ* stresses in the surrounding rock mass, and the rock material properties (Young's modulus E, Poisson ratio ν, yield strength σ_c, etc). It is also affected by the elastic properties and thickness t of the reinforced shotcrete lining installed to support the tunnel; this lining is the shaded area in Figures 1,2.

The design engineer is concerned to reduce cost by minimizing the volume of rock to be excavated, while keeping the stresses in the lining – and more particularly the bending moments through the lining thickness – within safety limits for the thickness and reinforcement used. The problem can thus be characterized as one of multi-objective optimization.

3. FINITE ELEMENT ANALYSIS

In a conventional design process, the engineer chooses the profile based on the rock properties and *in situ* stress field, using his/her experience (e.g. a low vertical stress in the rock permits a flatter base to the tunnel). The finite element method (FEM) is then used to model the construction numerically. The tunnel profile and rock geometry (e.g. strata, fractures) are used to generate a finite element mesh. The FEM takes as input the mesh (defined by nodal coordinates and connections), the material properties of the different soil or rock strata and the shotcrete, and the applied loads (including *in situ* stresses); it outputs the displacements of the nodes, and the stresses at the Gauss points in the elements [3]. If the stresses in the lining elements are within safety limits, the design is accepted.

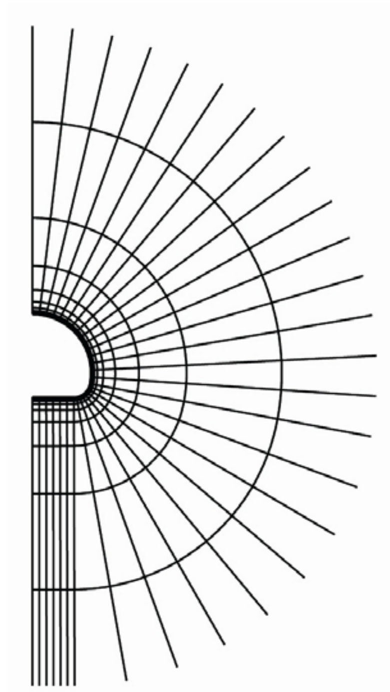

Figure 3: Typical FEM mesh (300 elements)

We have incorporated a FEM analysis within our genetic algorithm: the FELIPE [4] package combines 2D graphical pre- and post-processors with a suite of finite element 'main engines' written in Fortran77, for 2D elastic, elasto-viscoplastic and other applications. We have so far restricted attention to a homogeneous, isotropic elastic rock mass, and an in situ stress field defined by a vertical overburden stress σ_V and horizontal stress $\sigma_H = K_0.\sigma_V$ (K_0 is the lateral stress ratio). Figure 3 shows an example of a mesh generated within the GA program.

The FELIPE mesh uses eight-noded quadrilateral elements for the lining and rock mass, with mapped infinite elements where the rock extends beyond the mesh boundaries. There are four Gauss points in each element, and Figure 4 indicates how the Gauss-point stress values in the shotcrete lining (modeled by 3 layers of elements in this example) are extrapolated along a "spoke" AA through the lining to obtain the circumferential or hoop stresses on the inner and outer edges of the lining, σ_i and σ_o. These are used in the calculation of the fitness function (see section 4.2). If there are N_h elements around the tunnel, there will be $2N_h$ such "spokes".

4. THE FTO ALGORITHM

4.1 The GA chromosome

We showed in Section 2 that a tunnel profile is defined by the offset distance z , the upper radius r_1 , the ratio r_{21} and the angles α and β . The FTO program contains an algorithm to generate a graded finite element mesh using these five parameters, plus the tunnel lining thickness t , as illustrated in Figure 3. The FELIPE elasticity analysis then produces the Gauss-point stresses, which are used to evaluate the fitness function (see next section). This simulation forms the fitness-evaluation component of the FTO (FELIPE Tunnel Optimisation) genetic algorithm package.

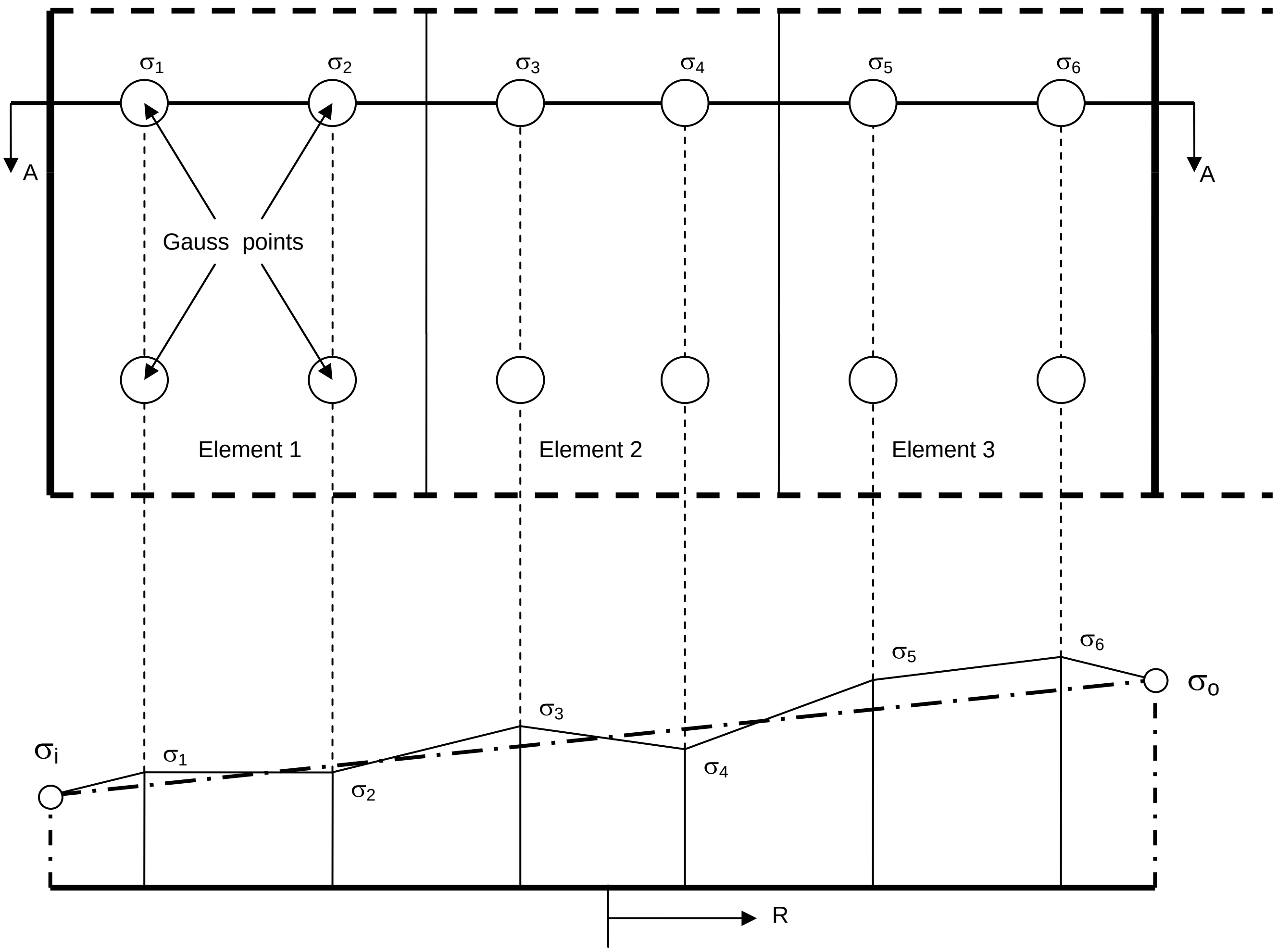

Figure 4: Extrapolating stresses in the shotcrete lining

Early experiments used such a five-parameter chromosome $\{r_1, r_{21}, \alpha, \beta, z\}$ to define individuals in the GA population; however, a serious problem arises. Although both parent chromosomes may define tunnels which enclose the prescribed minimum polygon $P_1 \ldots P_5$ in Figure 1, there is no guarantee that this property will be preserved after crossover or mutation.

The solution used in FTO is to define an individual chromosome using $\{r_{21}, \alpha, \beta, z\}$ only. In evaluating the fitness of this individual, the simulation first determines the minimum value of r_1 required in conjunction with these four parameter values in order to produce a tunnel profile enclosing the $P_1 \ldots P_5$ polygon as in Figure 1. This approach also ensures that each individual produces a tunnel profile which is close to the polygon (touching it at one or more places), and so of higher fitness than a tunnel which is unnecessarily large. The average quality of the next generation of tunnels is thus kept high.

4.2 The fitness function

The primary quantity determining the tunnel stability is the total bending moment M in the shotcrete lining. This is given by

$$M = \sum_{k=1}^{2N_h} \frac{1}{12} \Delta\sigma_k .bt^2 .d_k$$

where the summation is over the $2N_h$ Gauss-point "spokes" AA through the lining as shown in Figure 4, and

$$\Delta\sigma = \sigma_i - \sigma_o .$$

Here, t is the lining thickness, and b is the out-of-plane dimension (usually taken as 1 metre); these are constants for the tunnel. The mean distance between spoke k and its neighbours is denoted by d_k.

We seek to minimize M and also the area A enclosed by the tunnel (subject to enclosing the minimum polygon); this area can be calculated from the geometric construction in Figure 1. These parameters are combined in a function F:

$$F = f_A . A + f_M . M$$

where the weights f_A, f_M are chosen empirically. The fitness function to be maximized is then constructed as:

$$f(r_1, r_{21}, \alpha, \beta, z) = F_0 / (F_0 + F)$$

with a scaling parameter F_0.

4.3 Genetic operators

Two standard crossover operators for real-valued GAs, namely arithmetic crossover and discrete crossover [5, p. 190], have been employed. A nonuniform mutation of a single parent [5, p. 205] is also programmed. Each operator is associated with a probability of 0.32, together with a reproduction operator having a probability of 0.04. This latter probability is set low to prevent a relatively high-quality solution swamping the population with clones at an early stage of the run. Children from these genetic operations are subject to a 2% mutation rate.

A niche technique is also used to prevent premature convergence to a local optimum. Many such techniques have been developed – see for example [6,7]. The approach here is to cluster the initial population into "villages"; each village contains between 5 and 50 individuals (villagers) with very similar parameters. The minimum distance between villages decreases as the run proceeds. A member of a village can only perform crossover with another member of the same village, or with the current best individual in the population. Individuals not belonging to a village can perform crossover with any other individual.

4.4 Implementation

The main FTO program is written in Fortran95, compiled with the Salford compiler, and runs as a console application. Good results are obtained from a population of size 200, producing 200 children at each generation. Normally 30-50 generations, which corresponds to about 12000 fitness evaluations, are enough to reach convergence. A single fitness evaluation using a Pentium P4 PC takes about 230ms, and a whole run approximately 40 minutes. These timings depend greatly on the platform being used, and the size of the finite element meshes.

A separate program, FTOview, monitors the dynamically updated results file, and produces graphical output. It is programmed in C#, which is part of the new Microsoft .NET technology. Output from FTOview is contained in the next section.

5. RESULTS

To illustrate the application of FTO, we consider the following tunnel design problem. A tunnel is to be built at a depth of 55 metres, in rock with an overburden stress due to a rock density of 25 KN/m^3, and a lateral stress ratio $K_0 = 0.6$. The material elastic parameters are:

$$E = 5\times10^7 \text{ KN/m}^2 , \quad \nu = 0.25 \quad \text{(rock)}$$

$$E = 3\times10^7 \text{ KN/m}^2 , \quad \nu = 0.25 \quad \text{(shotcrete)}$$

The tunnel is required to enclose the polygon shown in Fig. 1(a) – maximum width 9.5 metres, and height 5.95 metres.

The weighting parameters f_A , f_M , F_0 used in the definition of the fitness function (Section 4.2) are 5.0, 1.0 and 500 respectively.

Figure 5 shows the population of random-parameter tunnel profiles in the initial generation of the GA. We recall that each individual is defined by the parameters $\{r_{21} , \alpha , \beta , z\}$, and the upper radius r_1 is calculated to ensure that the tunnel touches the minimum enclosed polygon at one or more points. However, many of the tunnels are completely impractical, enclosing large areas of "wasted space" outside the polygon.

Figure 6 shows the population after 5 generations, by which stage all the poor-quality individuals have disappeared. The corner sectors BEC and vertical offset distance OO' in the construction in Figure 2 are also shown. From these we see that there is still wide population diversity.

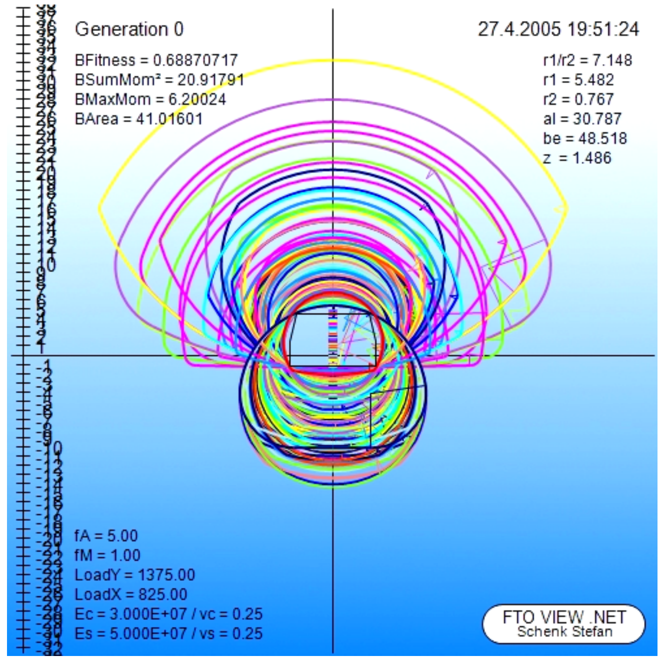

Figure 5: Initial population of tunnel profiles

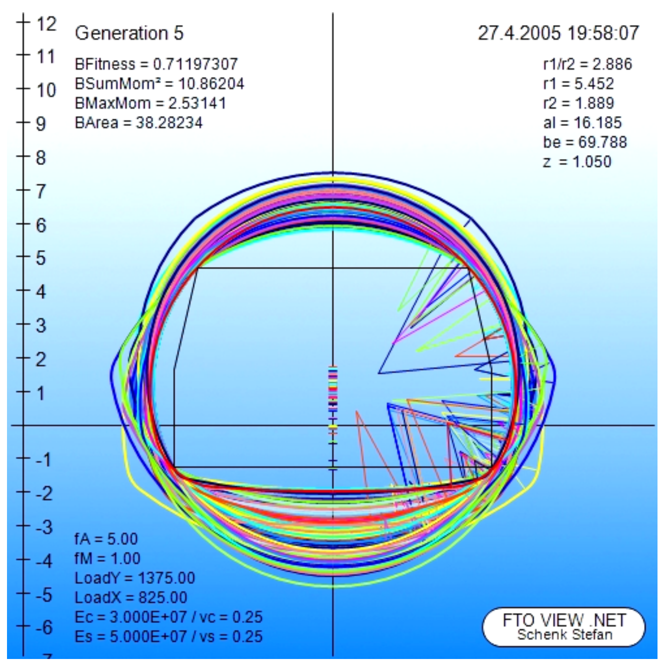

Figure 6: Population after 5 generations

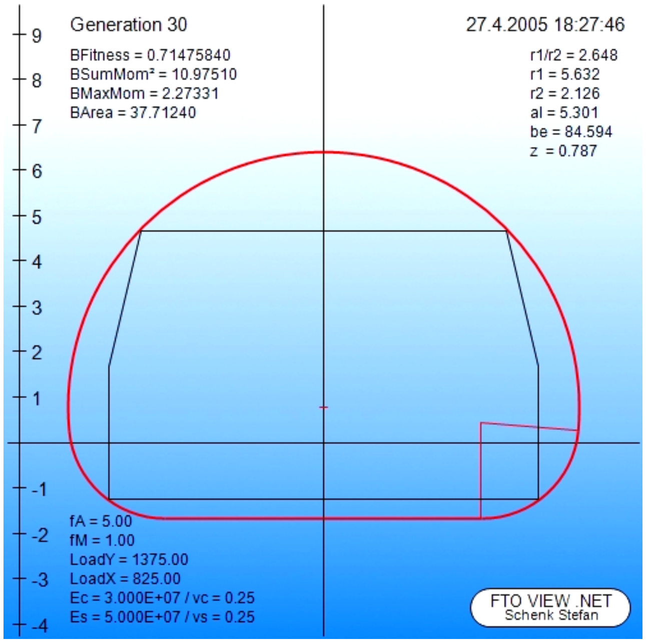

Figure 7: Final population (after 30 generations)

The final solution, reached after 30 generations, is shown in Figure 7. For comparison, Figure 8 shows the solution reached when a deeper-level tunnel is required (100m below the surface instead of 55m below). The higher in situ stresses result in an optimum profile which is more circular in shape. This accords with engineering experience.

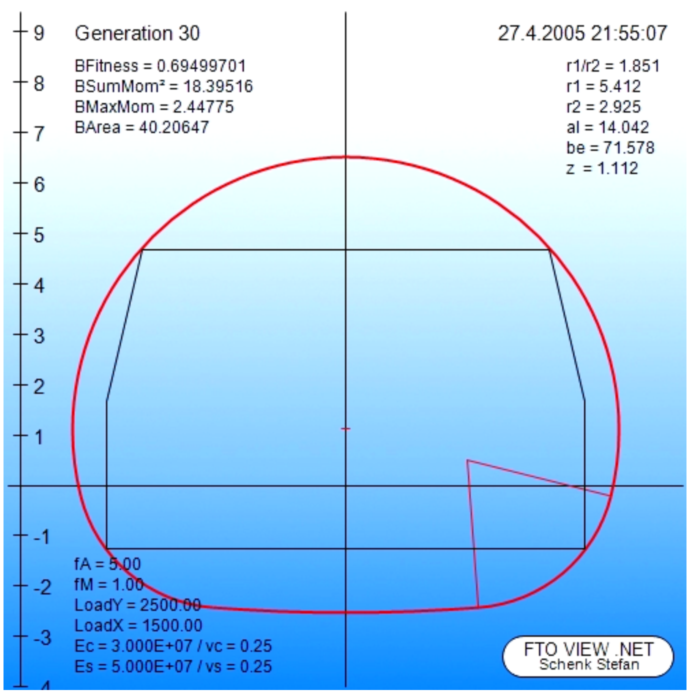

Figure 8: Tunnel at depth of 100 metres.

6. CONCLUSIONS

Genetic algorithms, coupled with finite element analyses for fitness evaluation, have proved highly successful in automating the design optimization process for underground tunnels. The empirical parameters in the fitness function (Section 4.2) can be adjusted to reflect the relative cost factors for excavation and reinforcement in the construction project. Experiments have found that the solutions found by the GA are in good accord with engineering judgement over a wide range of conditions (the *in situ* stress field, rock stiffness, etc).

Experiments have so far been limited to elasticity analyses; however, the practical use of this technique would be in models involving material or geometric nonlinearity, e.g. elasto-plastic or fractured rock.

Most of the CPU time required in a GA run has been taken by the finite element solutions; this aspect will become even more important when nonlinear models are used. These could be greatly speeded up by the use of iterative solution algorithms such as preconditioned conjugate gradients, in place of the direct frontal solver used here, since the solution obtained from the first mesh would provide a good starting-point for subsequent FEM solutions.

7. ACKNOWLEDGMENTS

The authors are grateful to the British Council, Vienna, and the Austrian Federal Science Ministry for funding a travel grant in association with this project. We also acknowledge the financial contribution of the University of Innsbruck.

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