Learning, Anticipation and Time–Deception in Evolutionary Online Dynamic Optimization

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ABSTRACT

In this paper we focus on an important source of problem–difficulty in (online) dynamic optimization problems that has so far received significantly less attention than the traditional shifting of optima. Intuitively put, decisions taken now (i.e. setting the problem variables to certain values) may influence the score that can be obtained in the future. We indicate how such time–linkage can deceive an optimizer and cause it to find a suboptimal solution trajectory. We then propose a means to address time–linkage: predict the future by learning from the past. We formalize this means in an algorithmic framework. Also, we indicate why evolutionary algorithms are specifically of interest in this framework. We have performed experiments with two new benchmark problems that contain time–linkage. The results show, as a proof of principle, that in the presence of time–linkage EAs based upon this framework can obtain better results than classic EAs that do not predict the future.

Categories and Subject Descriptors

F.1.2 [**Computation by Abstract Devices**]: Modes of Computation—*Online Computation*; G.1 [**Numerical Analysis**]: Optimization; I.2 [**Artificial Intelligence**]: Problem Solving, Control Methods, and Search

General Terms

Algorithms, Performance, Experimentation

Keywords

Evolutionary Algorithms, Dynamic Optimization, Online Optimization, Learning, Predicting

1. INTRODUCTION

The majority of the literature on dynamic optimization [11] involves the tracking of optima as the search space transforms over time. If evolutionary algorithms (EAs) [14] are used to achieve this goal, issues such as maintaining diversity around (sub)optima and continuously searching for new regions of interest that may appear over time are the most important [1, 2, 3, 4, 7, 8, 9, 13, 15, 16, 19, 24, 27, 31]. The shifting of optima in dynamic optimization problems is important to study and to (re)design EAs for. However, there is another feature of dynamic optimization problems that is common in real–world problems such as scheduling [10] and vehicle routing [21, 22, 29] that has received less attention in the literature. We will call this feature *time–linkage*.

Intuitively put, the presence of time–linkage in a dynamic optimization problem causes decisions that are made now, which are often made on the basis of maximizing a certain score right now, may influence the maximum score that can be obtained in the future. This in turn decreases the overall score obtained in the long run. A typical and illustrative example is the case of dynamic vehicle routing where the locations to visit are announced over time. If locations are clustered, but the clusters themselves are far apart, routing on the basis of the currently available locations will likely lead to oscillatory behavior of the vehicles if the announced locations oscillate between the clusters. More efficient routes could be formed by keeping vehicles inside clusters and only occasionally letting them move to another cluster. In addition, quality of service (e.g. being on time) as determined by the routing, influences future customer demand. Poor performance will likely not result in repeated orders. Hence, the revenue of a company over time will be determined by the current performance, but also by the impact the current way of routing has on future events.

Time–linkage in dynamic optimization problems is to a certain extent related to the temporal credit assignment problem in reinforcement learning [25]. In the temporal credit assignment problem, the feedback from the system for an action taken now is returned after a certain delay. Hence it is uncertain when taking an action what its eventual effect is on the environment. The problem then is how to assign credit to an action taken in the past based upon feedback received now. Ultimately, the goal in reinforcement learning is to learn a policy for navigating a state space optimally and the same action may be taken starting from the same state multiple times while performing reinforcement learning. The difficulty in temporal credit assignment is that actions and their corresponding rewards are not synchronized, which makes reinforcement learning more difficult. In essence, learning a policy is a static optimization problem where policies are solutions and the decisions taken during the process serve to aid in determining a good policy. This is fundamentally different from the dynamic optimization problems that we study in this paper where

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the decisions themselves must be made optimally and they can be made only once. What these two problems have in common is that in both cases the use of predictions of consequences of actions taken now can help to obtain better results [23]. Whereas this is already known for the temporal credit assignment problem in reinforcement learning, in this paper we show that this is also the case for time–linkage in dynamic optimization problems.

It is important to note that such dependencies between decisions over time requires their explicit processing in an algorithm to ensure the best performance in the long run. Any approach that does not explicitly anticipate these dependencies and instead only solves the problem for the current time will never obtain an optimal result.

In this paper, we present an algorithmic framework for solving dynamic optimization problems that is specifically equipped with the possibility of processing time–linkage. To this end, we propose the incorporation of learning (e.g. statistical [30] or machine [20]) with the explicit task of predicting the future to prevent being deceived over time. An evolutionary approach in which the future is predicted for dynamic optimization has been proposed before [28]. However, the cited approach only predicts the future for a single discrete timestep. As a result, the algorithm cannot process longer, arbitrary sized, time–linkage intervals. Moreover, the approach was only tested on a problem that doesn't contain the time–linkage aspect. As a result, no significant difference was observed in using either a good predictor or a bad predictor. In this paper, we present two new benchmark problems that contain time–linkage and show, as a proof of principle, how they can be solved using an instance of our proposed framework.

It should be noted that it is not our goal in this paper to propose a new state–of–the–art EA for dynamic optimization. Instead, we want to point out the influence that time–linkage can have and how, in a general manner, EAs can be equipped with tools to cope with time–linkage.

The remainder of this paper is organized as follows. In Section 2 we characterize online dynamic optimization problems. In Section 3 we discuss solving online dynamic optimization problems by only taking into account the current situation. Section 4 describes the advantages of solving online dynamic optimization problems by also taking into account future implications of decisions. In Section 5 we describe our algorithmic framework and in Section 6 we present results of running experiments with EAs based on this framework. Finally, possible directions for future research as well as conclusions are presented in Section 7.

2. ONLINE DYNAMIC OPTIMIZATION

2.1 Dynamic optimization

In general, optimization problems can be defined as:

$$\max_{\boldsymbol{\zeta}\in\mathbb{P}} \{\mathfrak{F}_{\boldsymbol{\gamma}}(\boldsymbol{\zeta})\} \textbf{ subject to } \mathfrak{C}_{\boldsymbol{\gamma}}(\boldsymbol{\zeta}) = \textit{feasible} \qquad (1)$$

where $\mathfrak{F}_{\boldsymbol{\gamma}} : \mathbb{P} \rightarrow \mathbb{O}$ is the optimization function, $\mathbb{P}$ is the parameter space, $\mathbb{O} = \mathbb{R}^{n_o}$ is the n_o–dimensional objective space, $\mathfrak{C}_{\boldsymbol{\gamma}} : \mathbb{P} \rightarrow \{\textit{feasible}, \textit{infeasible}\}$ is the constraint function and $\boldsymbol{\gamma} \in \mathbb{G}$ are problem–specific parameters.

In dynamic optimization the optimization function and the constraint function are functionals where the function space to optimize over consists of functions $\boldsymbol{\zeta}(t)$ of the time variable $t \in \mathbb{T} = [0, t^{\text{end}}]$, $t^{\text{end}} > 0$:

$$\mathfrak{F}_{\boldsymbol{\gamma}}(\boldsymbol{\zeta}(t)) = \int_0^{t^{\text{end}}} \mathfrak{F}^{\text{dyn}}_{\boldsymbol{\gamma}^{\text{dyn}}(t)}(\boldsymbol{\zeta}(t))\, dt \qquad (2)$$

$$\mathfrak{C}_{\boldsymbol{\gamma}}(\boldsymbol{\zeta}(t)) = \begin{cases} \textit{feasible} & \text{if } \forall t \in [0, t_{\text{end}}]\text{: } \mathfrak{C}^{\text{dyn}}_{\boldsymbol{\gamma}^{\text{dyn}}(t)}(\boldsymbol{\zeta}(t)) = \textit{feasible} \\ \textit{infeasible} & \text{otherwise} \end{cases}$$

where the convention that $\int_a^b (f_0(x), \ldots, f_{n_o-1}(x))\, dx = \left(\int_a^b f_0(x)dx, \ldots, \int_a^b f_{n_o-1}(x)dx\right)$ is used. Note that the dynamic variants of the optimization– and constraint function have parameters $\boldsymbol{\gamma}^{\text{dyn}}(t)$ that may change over time. Note that they may be dependent on earlier decisions by using the time variable indirectly, i.e. through a function of $\boldsymbol{\zeta}(t)$.

In the above time is assumed to be continuous. However, the parameters do not have to change continuously over time. In that case, the integral can be written as a discrete sum and the dynamic problem is said to be discrete.

2.2 The online case

Equation 2 can still be solved in an offline manner by searching for the best function $\boldsymbol{\zeta}(t)$ either in parametric or time–discretized form. It is the online variant or treatment of dynamic optimization that is the most interesting and the most practical, but unfortunately also the most difficult.

In the online case the dynamic optimization problem must be solved as time goes by. In other words, solutions cannot be evaluated for any future time $t > t^{\text{now}}$. Hence, decisions on what values to use for the variables at the current time t^{now} have to be made continuously. The only thing that can be evaluated is how well the algorithm has done so far, i.e. the goodness of the history and the present:

$$\mathfrak{H}^{\text{dyn}}(t^{\text{now}}, \boldsymbol{\zeta}(t)) = \int_0^{t^{\text{now}}} \mathfrak{F}^{\text{dyn}}_{\boldsymbol{\gamma}^{\text{dyn}}(t)}(\boldsymbol{\zeta}(t))\, dt \qquad (3)$$

2.3 System influence and control influence

We distinguish between two types of influence that cause the dynamic optimization problem to change with time:

1. *System influence.* This is the type of influence that the problem solver has no control over. It is the part of the dynamic system that changes over time, regardless of choices made for the problem variables. It is the inherent reason why the optimization problem is dynamic and hence why the optimization function parameters $\boldsymbol{\gamma}^{\text{dyn}}(t)$ are a function of time.
2. *Control influence.* This type of influence is the response of the dynamic system at time t^{now} to the choices of the problem variables made in the past, i.e. the trajectory $\boldsymbol{\zeta}(t)$ with $t \in [0, t^{\text{now}})$.

Most EAs designed for dynamic optimization problems studied in the literature are specifically designed to cope with system influences. Without taking into account the (possible) presence of control influence however, the online dynamic optimizer risks falling victim to time–deception.

3. OPTIMIZING PRESENT: VICTIM TO TIME–DECEPTION

3.1 The approach

An often–used approach to solving online dynamic optimization problems is to optimize $\mathfrak{H}^{\text{dyn}}(t^{\text{now}}, \boldsymbol{\zeta})$ continuously or whenever an event resulting from system influence takes

place. Since we cannot change the past, we can only vary the settings of the variables at t^{now}. Hence, the optimization problem to solve at time t^{now} using this approach is actually static: optimize the value of the dynamic optimization function at time t^{now}. To cope with a variety of system influences when using EAs, diversity preserving mechanisms are often used to prevent complete convergence as are other techniques such as detecting (major) changes in the landscape to trigger a restart or forking off multiple sub–populations from a general optimizer to search various parts of the search space more closely as they become more interesting over time.

3.2 How bad can it be?

Unfortunately, the answer is *arbitrarily bad*. The most important reason for this is the presence of control influence. Of course system influence could make the problem change in a random way, clearly already making the problem arbitrarily difficult. However, even if system influence is smooth and the landscape is not–complex, optimizing only the current situation can lead to arbitrarily bad results. Consider for instance the following unconstrained l–dimensional dynamic optimization problem; a simple adaptation of the sphere problem that shifts with time:

$$\max_{\boldsymbol{\zeta}(t)} \left\{ \int_0^{t^{\text{end}}} \varphi(\boldsymbol{\zeta}(t), t) dt \right\} \tag{4}$$

where

$$\varphi(\boldsymbol{\zeta}(t), t) = \begin{cases} -\sum_{i=0}^{l-1} \left(\boldsymbol{\zeta}(t)_i - t\right)^2 & \text{if } 0 \leq t < 1 \\ -\sum_{i=0}^{l-1} \left(\boldsymbol{\zeta}(t)_i - t\right)^2 + \psi\left(\left|\boldsymbol{\zeta}(t-1)_i\right|\right) & \text{otherwise} \end{cases}$$

Now, when optimizing only the present in an online setting, a value for $\boldsymbol{\zeta}(t^{\text{now}})$ is chosen by maximizing $\varphi(\boldsymbol{\zeta}, t^{\text{now}})$. But for any t, $\varphi(\boldsymbol{\zeta}, t)$ is just a hyperparabola with a unique maximum for $\boldsymbol{\zeta}(t)_i = t$. For $0 \leq t < 1$ the associated value is 0 and for $t \geq 1$ this value is $-\sum_{i=0}^{l-1} \psi\left(\left|\boldsymbol{\zeta}(t-1)_i\right|\right)$. It is this construction that deceives an approach in which only the present is optimized because then the actual value of function $\psi(\cdot)$ is not taken into account although it may decrease at an arbitrary rate, depending on its form.

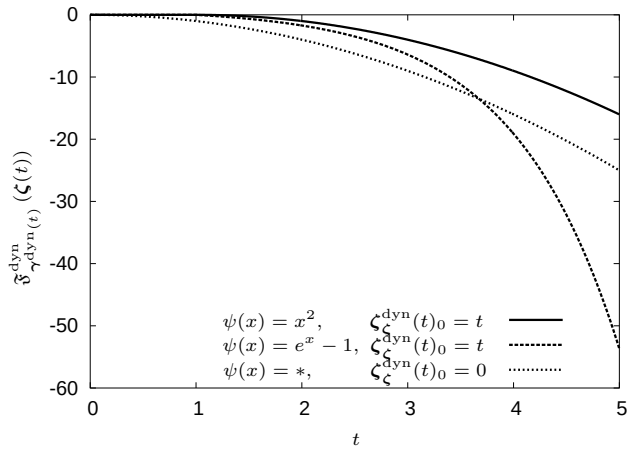

Figure 1: Illustration of the optimization values obtained for different variable trajectories and different forms of $\psi(\cdot)$ in the case of $l = 1$ and $t^{\text{end}} = 5$.

If however $\boldsymbol{\zeta}(t)_i = 0$ is simply always chosen, then, assuming that $\psi(0) = 0$, the optimization value that is reached is $l \int_0^{t^{\text{end}}} -t^2 dt = -\frac{l}{3}\left(t^{\text{end}}\right)^3$, regardless of function $\psi(\cdot)$.

Now if for instance $\psi(x) = x^2$, the result is better if only the present is optimized than if just $\boldsymbol{\zeta}(t)_i = 0$ is chosen. Although a better result can still be obtained because $\boldsymbol{\zeta}(t)_i = 0$ is not the optimal solution, the penalty of time–deception is only small. But if $\psi(\cdot)$ is a higher–order increasing function, such as for instance $\psi(x) = e^x - 1$, a (much) worse optimization value will be obtained. A graphical illustration of the difference in obtained optimization values for different variable trajectories for $l = 1$ is given in Figure 1.

Since in the online case the behavior of the optimization function in the future is not known, optimizing only the present can thus significantly reduce overall solution quality. Hence, optimizing only the present is not a good approach unless it the problem is provably not time–deceptive.

4. OPTIMIZING PRESENT AND FUTURE: LEARN TO AVOID TIME–DECEPTION

4.1 The approach

The approach of optimizing only the present is deceived over time because the true problem definition (i.e. equation 2) is not used. Future changes that occur as a result of decisions made earlier are neglected. To remedy this, optimization over future choices is required. In the online case however, an evaluable future is absent. Hence the only option is to *predict* the future. The available information to base that prediction upon besides problem–specific information is information collected in the past. The better the prediction, the closer the algorithm can get to optimality.

Summarizing, the optimization problem to be solved using this approach at any time t^{now} is based on an approximation of the value of the dynamic optimization function over a future timespan of length t^{plen}:

$$\max_{\boldsymbol{\zeta}(t)} \left\{ \int_{t^{\text{now}}}^{\min\left\{t^{\text{now}}+t^{\text{plen}}, t^{\text{end}}\right\}} \hat{\mathfrak{F}}^{\text{dyn}}_{\boldsymbol{\alpha}}(t, \boldsymbol{\zeta}(t))\, dt \right\} \tag{5}$$

$$\textbf{s.t. } \forall t \in \left[t^{\text{now}}, \min\left\{t^{\text{now}}+t^{\text{plen}}, t^{\text{end}}\right\}\right] : \hat{\mathfrak{C}}^{\text{dyn}}_{\boldsymbol{\alpha}}(t, \boldsymbol{\zeta}(t)) = \textit{feasible}$$

where $\hat{\mathfrak{F}}^{\text{dyn}}_{\boldsymbol{\alpha}}\left(t^{\text{now}}, \boldsymbol{\zeta}(t^{\text{now}})\right) = \mathfrak{F}^{\text{dyn}}_{\boldsymbol{\gamma}^{\text{dyn}}(t^{\text{now}})}\left(\boldsymbol{\zeta}(t^{\text{now}})\right)$

4.1.1 Prediction in the complete BBO case

The complete BBO (Black–Box Optimization) case is the most general case. No prior knowledge on the problem to be solved is assumed other than the number of variables and their types. Additional knowledge can only be gained by evaluating solutions. Since nothing is known about the optimization function, only a a very general form of induction can be performed to predict future function values.

We assume that the number of variables and their semantics do not change. To predict the (expected) value of the dynamic optimization function, an approximation based on previously evaluated solutions can be used. Computing this approximation is a (statistical) learning problem. The available data in the learning problem is:

$$\bigcup_{i=0}^{n_{\text{data}}-1} \left\{ \left(\left(t^i, \boldsymbol{\zeta}^i\right), \boldsymbol{y}^i \right) \right\} \tag{6}$$

where t^i, $0 < t^i \leq t^{\text{now}}$ is the time–component of the i–th pattern in the data set, $\boldsymbol{\zeta}^i$ is the variable–value–component and $\boldsymbol{y}^i$ contains the value of the dynamic optimization function for $\boldsymbol{\zeta}^i$ at time t^i. Note that the use of an EA can greatly add to the availability of data and can hence increase the accuracy of the predictions because a (diverse) population is used. Each population member can serve as a pattern.

The actually chosen trajectory $\boldsymbol{\zeta}(t), t \in [0, t^{\text{now}})$ is of course also available to the predictor. Parts of this history–trajectory can be integrated into the dataset to be able to process time–linkage, i.e. dependencies of the dynamic optimization function on values actually chosen for the problem variables in the past.

The goal of learning is to estimate the value of the dynamic optimization function for future times (assuming that the constraint function does not need to be estimated) by minimizing the generalization error over the timespan that contains the data to learn from. In the single–objective case:

$$\min_{\boldsymbol{\alpha} \in \mathbb{A}} \left\{ \int_{t^{\min}}^{t^{\max}} \left(\mathfrak{F}^{\text{dyn}}_{\gamma^{\text{dyn}}(t))} \left(\boldsymbol{\zeta}(t) \right) - \hat{\mathfrak{F}}^{\text{dyn}}_{\boldsymbol{\alpha}} \left(\boldsymbol{\zeta}(t) \right) \right)^2 dt \right\} \quad (7)$$

where

$$\begin{cases} t^{\min} &= \min_{i \in \{0,1,\ldots,n_{\text{train}}-1\}} \{t^i\} \\ t^{\max} &= \max_{i \in \{0,1,\ldots,n_{\text{train}}-1\}} \{t^i\} \end{cases}$$

and $\boldsymbol{\alpha} \in \mathbb{A}$ are the parameters of the function class from which to choose the approximation.

4.1.2 Prediction in the partial BBO case

In the presence of problem–specific information, the learning task may be less involved which may improve the reliability of the predictions. A typical case is when the function can be evaluated for any $0 \leq t < t^{\text{end}}$, as long as the required parameters are set. Then, if we are able to predict the values for the parameters accurately, we automatically get an accurate function evaluation. The less parameters to estimate, the better the hope of obtaining good approximations.

4.1.3 Prediction length and prediction base

Two key issues are how far into the future predictions should be made (prediction length, denoted t^{plen}) and information from how far in the past should be used to base the prediction upon (prediction base, denoted t^{pbase} indicating generally collected data and history length, denoted t^{hlen} indicating the history of the actually chosen trajectory). A proper choice for the prediction length and the history length depends on the time–linkage timespan, i.e. how far into the future do current choices have a significant influence? Certainly this is also the minimal choice for the prediction base. However, larger values for prediction length, prediction base and history length may be required to look beyond the deception and observe the general dynamics of the optimization problem.

Another issue that influences the proper choice for the prediction length is the reliability of predictions. As predictions are made further into the future, they are bound to become less reliable, giving a trade–off between the required prediction length as a result of time–linkage and the feasible prediction length as a result of reliability issues.

4.2 How good can it be?

Fortunately, the answer is *arbitrarily good*. However, although it is intuitively clear that the optimum is attainable, this does require *perfect* predictions. Then, the problem can be solved to optimality by optimizing at any time the integral over the predictions with $t^{\text{plen}} = t^{\text{end}} - t^{\text{now}}$.

The strength of the optimization method (with respect to the problem at hand) is still a key component to success. However, the success of the approach now also heavily depends on the strength of the prediction method. Bad predictions may even lead to worse results than are obtained by optimizing only the present. Hence, careful design and performance assessment of methods that predict the future are certainly called for. In the following section we present a general framework for solving dynamic optimization problems by incorporating learning techniques as described above.

5. ALGORITHMIC FRAMEWORK

5.1 Components

5.1.1 Solver

The solver, denoted S, is an optimization algorithm, possibly equipped with tools to allow for adaptability as time changes. The function to be optimized is provided by the function component discussed in Section 5.1.3.

5.1.2 Predictor

The predictor, denoted P, is a learning algorithm that approximates either the optimization function directly or several of its parameters. The data set from which to estimate a function is provided by the database component discussed in Section 5.1.4. When called upon, the predictor returns either the predicted function value directly or predicted values for parameters.

5.1.3 Function

The function, denoted F, is the optimization function to be maximized by the solver. If the future is not to be taken into account, this function is just the dynamic optimization function. The trajectory of the variables in the predicted future represents the variables to be optimized by the solver. To be able to compute the optimization value of such a future trajectory, a solution for each possible time between t^{now} and $\min\{t^{\text{now}} + t^{\text{plen}}, t^{\text{end}}\}$ is needed. It is convenient to divide the trajectory–future interval as well as the trajectory–history interval into non–zero sub–intervals of length t^{pint} and t^{hint} respectively. The optimization value then is a discrete approximation of the integral over the future interval where future predictions are in addition to other data based on a discretized past trajectory. The number of variables that the solver needs to optimize over is the union of all sets of variables that pertain to the dynamic optimization function at the beginning of the sub–intervals. The dynamic optimization function is used to compute the optimization value that pertains to the current time and the predictor is used to predict the future.

5.1.4 Database

The database, denoted D, is a collection of patterns upon which the predictor bases its predictions. Patterns are added either by the function component (i.e. in the complete BBO case whenever a new solution is evaluated) or by the system

whenever an event occurs that is related to the parameters of interest (i.e. in the partial BBO case). All patterns are time–stamped. The database only contains patterns with a timestamp t for which $t^{\text{now}} - t^{\text{pbase}} \leq t \leq t^{\text{now}}$ holds.

5.1.5 Timer

The timer, denoted T, can provide the current time t^{now}.

5.2 Dividing resources

Clearly, optimization becomes more involved if we also want to take into account predictions of the future. Not only does the number of variables to optimize over increase (at least if we regard the complete BBO case), but also additional time is required for learning to make predictions.

It is important to note that there is a trade-off between how much time should be spent on running the solver and how much time should be spent on running the predictor. To allow for a scheme that implements this trade–off we propose to implement the solver and the predictor components as threads. This allows both for a scheme in which the solution component and the predictor component run simultaneously as well as a scheme where the predictor and solver are run sequentially by synchronization using signals. An example of the second scheme is when the solver sends a signal when a certain number of generations have passed and subsequently awaits completion of the learning task before continuing.

5.3 Definition

To complete the framework, in this section we provide an algorithmic description of how the components are used together to solve online dynamic optimization problems. First, the trajectory is made empty and all the components are initialized. Then, the solver and the predictor are started and the actual optimization begins. Although the solver may store a solution into the trajectory at any time (e.g. at the end of each generation for an EA), we want to ensure that at least a few solutions are stored in the trajectory. To this end, the solver is requested for a solution at regular intervals of length t^{sint}. These requests are issued until t^{end} is reached. Then, the solver and the predictor are halted and the resulting trajectory is returned:

FRAMEWORK$(S, P, F, D, T, t^{\text{end}}, t^{\text{pbase}}, t^{\text{plen}}, t^{\text{sint}}, t^{\text{pint}})$

1 $\boldsymbol{Z} \leftarrow ()$
2 S.INITIALIZE$(S, P, F, \ldots, t^{\text{pint}})$
3 P.INITIALIZE$(S, P, F, \ldots, t^{\text{pint}})$
4 F.INITIALIZE$(S, P, F, \ldots, t^{\text{pint}})$
5 D.INITIALIZE$(S, P, F, \ldots, t^{\text{pint}})$
6 T.INITIALIZE$(S, P, F, \ldots, t^{\text{pint}})$
7 S.START()
8 P.START()
9 **do**
  9.1 $t^{\text{now}} \leftarrow T$.GETTIME()
  9.2 $\boldsymbol{\zeta} \leftarrow S$.REQUESTSOLUTION()
  9.3 $\boldsymbol{Z} \leftarrow \boldsymbol{Z} \sqcup ((\boldsymbol{\zeta}, t^{\text{now}}))$
  9.4 $t^{\text{next}} \leftarrow \min\{t^{\text{next}} + t^{\text{sint}}, t^{\text{end}}\}$
  9.5 AWAITTIME(t^{next})
  while $t^{\text{now}} \leq t^{\text{end}}$
10 S.STOP()
11 P.STOP()
12 **return**$(\boldsymbol{Z})$

The final part of the framework that is of a specific form is the way in which a solution in the form of a future trajectory is evaluated. It is here that the prediction component can influence the way in which the solver searches for a solution at t^{now} because the predictor is used to evaluate all parts of the trajectory that pertain to future times:

F.EVALUATE$((\boldsymbol{\zeta}^0, \boldsymbol{\zeta}^1, \ldots, \boldsymbol{\zeta}^{\lceil t^{\text{plen}}/t^{\text{pint}} \rceil - 1}))$

1 $t^{\text{now}} \leftarrow T$.GETTIME()
2 $\boldsymbol{y} \leftarrow t^{\text{pint}} \mathfrak{F}^{\text{dyn}}_{\boldsymbol{\gamma}^{\text{dyn}}(t^{\text{now}}, Z(t^{\text{now}}, \boldsymbol{\zeta}))}(\boldsymbol{\zeta}^0)$
3 **if** COMPLETEBBOCASE() **then**
  3.1 D.ADDPATTERN$(((\boldsymbol{\zeta}^0, t^{\text{now}}), \boldsymbol{y}))$
  3.2 **for** $i \leftarrow 1$ **to** $\lceil t^{\text{plen}}/t^{\text{pint}} \rceil - 1$ **do**
    3.2.1 $\boldsymbol{y} \leftarrow \boldsymbol{y} + t^{\text{pint}} P$.PREDICT$(\boldsymbol{\zeta}^i, t^{\text{now}} + i \cdot t^{\text{pint}})$
4 **else**
  4.1 **for** $i \leftarrow 1$ **to** $\lceil t^{\text{plen}}/t^{\text{pint}} \rceil - 1$ **do**
    4.1.1 $\boldsymbol{\gamma}^{\text{predicted}} \leftarrow P$.PREDICT$(\boldsymbol{\zeta}^i, t^{\text{now}} + i \cdot t^{\text{pint}})$
    4.1.2 $\boldsymbol{y} \leftarrow \boldsymbol{y} + t^{\text{pint}} \mathfrak{F}^{\text{dyn}}_{\boldsymbol{\gamma}^{\text{predicted}}}(\boldsymbol{\zeta}^i)$
5 **return**$(\boldsymbol{y})$

6. EXPERIMENTS

6.1 EA

The optimization problems that we use are real–valued, but at any point in time not very daunting as the most important thing we focus on is time–linkage. Therefore, we opt for a simple and fast real–valued EA. We use an EDA (Estimation–of–Distribution Algorithm) for real–valued optimization [5, 18] without learning dependencies between problem variables. The main difference with traditional EAs is that in EDAs a probabilistic model is learned using the selected solutions. The probabilistic model can capture various properties of the optimization problem. By drawing new solutions from the probabilistic model these properties can be exploited to obtain more efficient optimization.

In this paper we performed experiments with a real–valued EDA based on the normal distribution in which each variable is taken to be independent of all the other variables. Such an EDA is also known as the naive IDEA (Iterated Density–Estimation Evolutionary Algorithm) [6]. In the naive variant the mean and standard deviation of a one–dimensional normal distribution are estimated from the selected solutions for each variable separately. A new solution is constructed by sampling one value per variable from the associated one–dimensional normal distribution. Since the optimization problem is dynamic, we prevented total premature convergence by bounding the estimated variance for each variable to a minimum of 0.1. Finally, all results were averaged over 100 independent runs.

6.2 BBO: Time–deceptive numerical problem

6.2.1 The problem

We first investigate the real–valued time–deceptive problem introduced in Section 3, Equation 4. We regard two variants by setting $\psi(x) = x^2$ and $\psi(x) = e^x - 1$. Moreover, we have used a dimensionality of $l = 1$.

6.2.2 Instantiating the framework

We used three different predictor instances. In this problem the goal of the predictor is to predict the value of the optimization function directly. The first instance is optimal, i.e. it is the true value of the dynamic optimization function. The second instance estimates a linear function and the third instance estimates a quadratic function. The latter two estimates are computed using a least–squares approximation. For clarity: the linear function for example is estimated from a set of patterns with timestamps at most t^{pbase} time ago. Each pattern $((t^i, \boldsymbol{\zeta}^i), \boldsymbol{y}^i)$ is complemented with the actually chosen trajectory in the past up to time $t^i - t^{\text{hlen}}$ in steps of t^{hint}, i.e. if $t^{\text{hlen}} = t^{\text{hint}} = 1$, the pattern is transformed to $((t^i, \boldsymbol{\zeta}^i, \boldsymbol{\zeta}(t^i - 1)), \boldsymbol{y}^i)$. The linear estimator now is constructed from the transformed dataset.

Only the function class used by the quadratic estimator contains the target function for the case of $\psi(x) = x^2$. Hence, effective future predictions are possible with proper estimations in this case, preventing time–deception. For the case of $\psi(x) = e^x - 1$, neither of the estimators can represent the target function. However, the quadratic estimator should be capable of far better approximations.

Since we present a proof–of–principle, we do not investigate the selection of t^{plen} during optimization. Instead, we fix it to either 0 that corresponds to the traditional approach of not looking into the future or to the optimal value of 1. Moreover, we set $t^{\text{hlen}} = t^{\text{hint}} = t^{\text{pbase}} = t^{\text{pint}} = t^{\text{plen}}$.

6.2.3 Results

A population size of 25 was experimentally found to be adequate for solving the optimization problem in each time step. We set $t^{\text{end}} = 10$ and advanced time by a timestep of 0.001 every 25 evaluations (i.e.every generation). Since the database contains all patterns over a timespan of length 1 and the timesteps are of size 0.001, the size of the database can become quite large. Although this allows for a higher precision of estimations, it also results in large time requirements for the learning task. Learning was performed after a predefined number of generations had passed. To investigate the impact on the overall quality of optimization, we performed experiments with various values for the number of generations between learning phases: $1, 10, 100$ and 1000.

The average trajectories obtained for the quadratic and the exponential time–deceptive numerical problem are shown in Figures 2 and 3 respectively. The overall result (i.e. the integrated function over $t \in [0, 10]$) is tabulated in Table 1.

Theoretically, under the assumption that the length of the time–linkage is known, the optimal trajectory can be obtained if the target function is in the function class used by the learner and the learner is competent in that it will indeed find that target when learning. In the case of the quadratic time–deceptive numerical problem this is experimentally verified by the results. The use of the quadratic estimator leads to results that are very close to optimality (i.e. when the future is known). The discrepancy is explained by the startup time the learner needs before being able to construct a model based upon previously encountered data. Moreover, results improve if learning is performed more frequently because the model is then constructed earlier.

In the case of the exponential time–deceptive numerical problem, neither the linear nor the quadratic estimator provide a function class that contains the target exponential function. However, for the time–linkage in this problem that depends only on a single point in the past over a distance of 1, a quadratic function can quite closely approximate an exponential function. For this reason the use of the quadratic estimator leads to good results here as well, albeit not optimal. Small deviations from the optimal trajectory as a result of a small learner error can indeed be seen in Figure 3. The linear estimator is not capable of approximating a quadratic function well. For the exponential function, linear estimation is even worse. The use of the linear estimator therefore leads to far worse results. An even more important point to note is that the results using the linear estimator can be even worse than when prediction is not used because of the large errors in the predictions. Hence, another important issue in using learning for online dynamic optimization is the assessment of the reliability of predictions and the use of predictions only if this reliability is large enough.

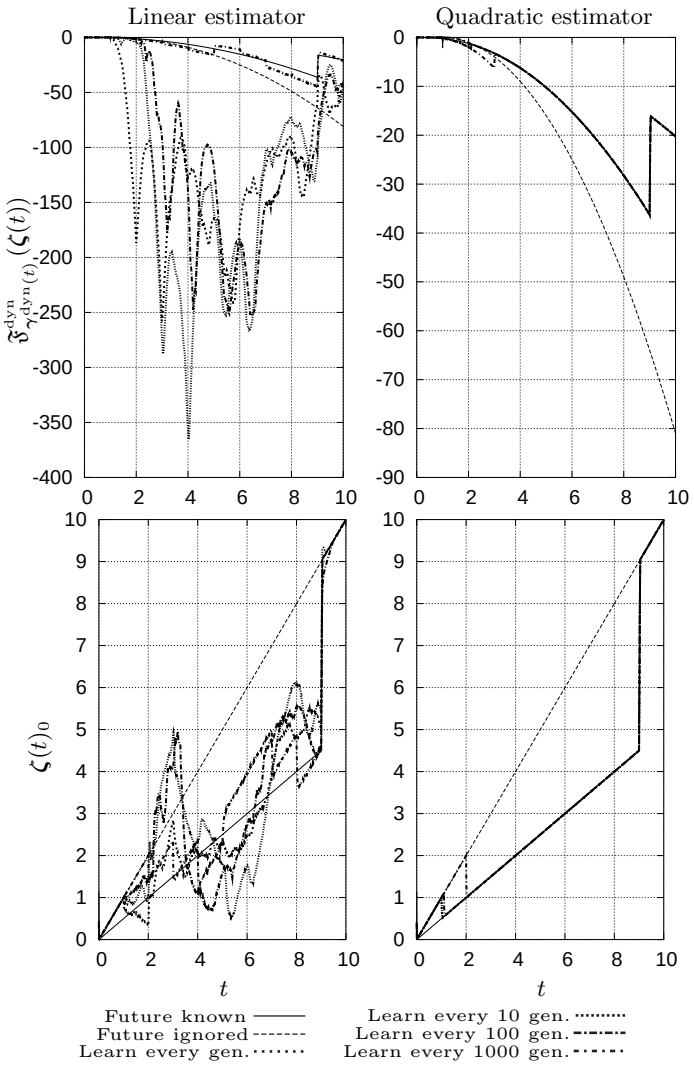

Figure 2: Results averaged over 100 runs on the time–deceptive numerical problem with $\psi(x) = x^2$.

The results lead to the expected conclusion that competent learners are called for and that reliability of predictions is a major issue. The competence of the learner in the BBO case depends on general/overall competence which is very hard to obtain. In the problem–specific case however, achieving learner competence may be easier because the shape of the model to be learned (i.e. parametric learning) is known from domain knowledge, ensuring that the target function is in the function class used by the learner.

6.3 Partial BBO: dynamic pickup problem

6.3.1 The problem

The second problem that we investigate is a discrete partial BBO problem. Although it is based on a very simple model, the time–linkage in the problem is large: any decision made now influences the result of the dynamic optimization function for all future timesteps. The intuitive description is that at timestep t a truck is located at $\boldsymbol{x}^{\text{truck}}(t)$ and a package appears at location $\boldsymbol{x}^{\text{package}}(t)$. It must now be decided whether to send the truck to go and pick up the package or to drive elsewhere. If the package is not picked up, it disappears. Picking up the package pays a value of 1, but

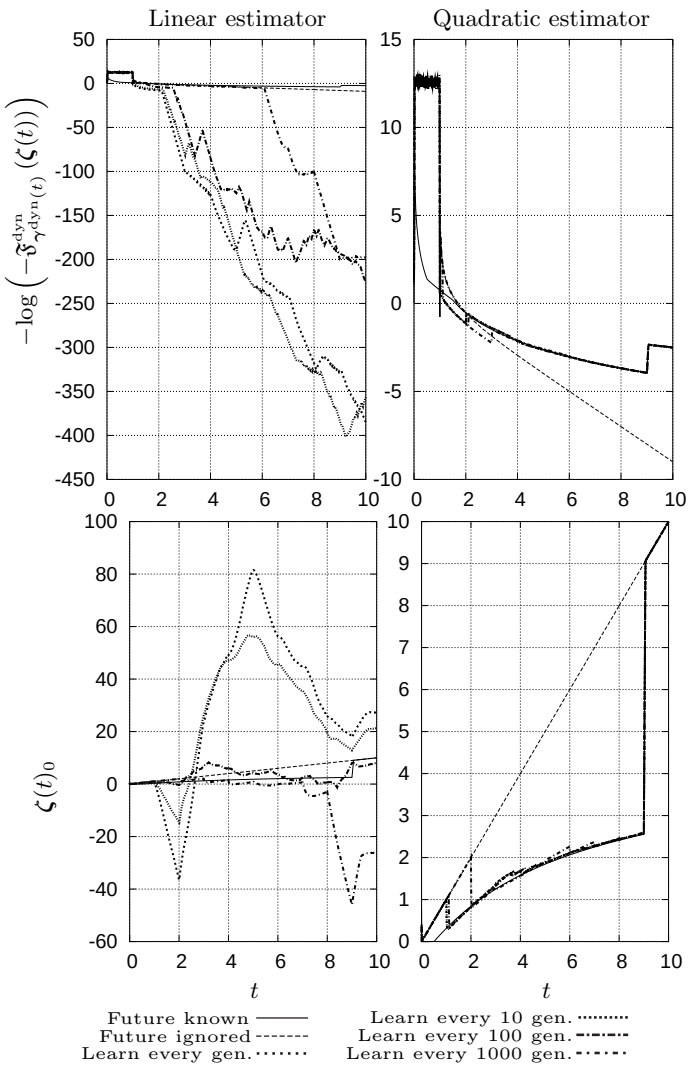

Figure 3: Results averaged over 100 runs on the time–deceptive numerical problem with $\psi(x) = e^x - 1$.

driving costs a value equal to the Euclidean distance traveled. The number of packages is $n_{\text{packages}} = t^{\text{end}} + 1$, i.e. the timesteps are of size 1. A solution at time t now is a tuple $\boldsymbol{\zeta}(t) = (b(t), \boldsymbol{x}^{\text{alternative}}(t))$ where $b \in \{0, 1\}$ indicates whether the package at time t should be picked up ($b(t) = 1$) and $\boldsymbol{x}^{\text{alternative}}(t)$ is the location to drive to if the package is not to be picked up ($b(t) = 0$). Mathematically:

$$\mathfrak{F}^{\text{dyn}}_{\gamma^{\text{dyn}}(t)}\left((b(t), \boldsymbol{x}^{\text{alternative}}(t))\right) = \tag{8}$$

$$\begin{cases} 1 - \parallel \boldsymbol{x}^{\text{package}}(t) - \boldsymbol{x}^{\text{truck}}(t) \parallel & \text{if } b(t) = 1 \\ 0 - \parallel \boldsymbol{x}^{\text{alternative}}(t) - \boldsymbol{x}^{\text{truck}}(t) \parallel & \text{otherwise} \end{cases}$$

where

$$\boldsymbol{x}^{\text{truck}}(t) = \begin{cases} \sim \prod_{i=0}^{l-1} \mathcal{N}(0,1) & \text{if } t = 0 \\ \boldsymbol{x}^{\text{package}}(t-1) & \text{if } t = 1 \text{ and } b(t-1) = 1 \\ \boldsymbol{x}^{\text{alternative}}(t-1) & \text{otherwise} \end{cases}$$

For simplicity, the model used to generate new package locations is a univariately factorized normal distribution with zero mean and unit variance, i.e. $\boldsymbol{x}^{\text{package}}(t) \sim \prod_{i=0}^{l-1} \mathcal{N}(0,1)$.

		$\psi(x) = x^2$	$\psi(x) = e^x - 1$
	Future known	$-1.21846 \cdot 10^2$	$-1.55430 \cdot 10^2$
	Future ignored	$-2.42940 \cdot 10^2$	$-8.08692 \cdot 10^3$
Linear	**Learn every gen.**	$-1.09434 \cdot 10^3$	$-2.04922 \cdot 10^{65}$
Linear	**Learn every 10 gen.**	$-1.18946 \cdot 10^3$	$-7.93853 \cdot 10^{172}$
Linear	**Learn every 100 gen.**	$-9.98665 \cdot 10^2$	$-3.02553 \cdot 10^{96}$
Linear	**Learn every 1000 gen.**	$-1.38907 \cdot 10^2$	$-1.22634 \cdot 10^{86}$
Quadratic	**Learn every gen.**	$\mathbf{-1.22010 \cdot 10^2}$	$\mathbf{-1.55966 \cdot 10^2}$
Quadratic	**Learn every 10 gen.**	$-1.22013 \cdot 10^2$	$-1.55969 \cdot 10^2$
Quadratic	**Learn every 100 gen.**	$-1.22062 \cdot 10^2$	$-1.56069 \cdot 10^2$
Quadratic	**Learn every 1000 gen.**	$-1.23178 \cdot 10^2$	$-1.58092 \cdot 10^2$

Table 1: Overall results (i.e. $\int_0^{t^{\text{end}}} \mathfrak{F}^{\text{dyn}}_{\gamma^{\text{dyn}}(t)}(\boldsymbol{\zeta}(t))\, dt$) on the time–deceptive problem.

6.3.2 *Instantiating the framework*

A very simple strategy is given by a hillclimber. The decision taken at each timestep is to move only to pick up a package and moreover only to do so if the distance to the package is less than 1. In other words, a negative score is never accepted.

We have compared the hillclimber with an EA instance of the general framework. Since the optimization function is completely known with the exception of $\boldsymbol{x}^{\text{package}}(t)$, the problem is only partially a BBO problem. Hence, we can restrict the prediction task to predicting future values for $\boldsymbol{x}^{\text{package}}(t)$. We have performed experiments where we assumed the distribution of $\boldsymbol{x}^{\text{package}}(t)$ to be known and where we estimated this distribution from data, assuming only that the data is indeed normally distributed.

In theory, the influence of any decision at time t influences the outcome of the dynamic optimization function at any time $t' > t$. However, the larger $t' - t$, the smaller the remaining impact on the situation at time t'. Although in theory it would be optimal to set t^{plen} to t^{end} with $t^{\text{pint}} = 1$, such a choice gives rise to two practical problems. First, a large t^{plen} gives to extremely large trajectories to optimize. This drastically increases the resources required by the EA to solve the problem. Second, since the future is inherently stochastic, a proper estimation of the expected future profits requires averaging evaluation over multiple calls. Moreover, the variability of these outcomes increases as t^{plen} increases because more uncertainty is introduced. Hence, unless an infinite number of calls is used, a smaller value for t^{tplen} is expected to be optimal in practice.

To prevent large trajectories to be subject to evolution in the EA, we choose a simplified approach by choosing a very special form for the predictor. The predictor predicts the result of the dynamic stochastic optimization function up to a timespan of t^{plen} into the future by using the hillclimber instead of explicit solutions for each timestep provided by the EA. The EA only provides a solution for the current time. Although better results may be obtained by allowing the EA also to evolve future solutions, using the hillclimber to predict the future can already give good solutions. The reason is that using the hillclimber can already give a good impression of the quality of a certain starting point. Since the dynamic optimization function is stochastic, it should be noted that multiple calls are required to estimate the expected future payoff even when evaluating the future using the hillclimber. To reduce the number of statistical errors, the best evolved decision is compared to the default choice of doing nothing, i.e. $b(t^{\text{now}}) = 0$ and $\boldsymbol{x}^{\text{alternative}}(t^{\text{now}}) =$

$\boldsymbol{x}^{\text{truck}}(t^{\text{now}})$. Only if the mean fitness of the best evolved decision averaged over 100 calls to the dynamic optimization function is statistically significantly larger than the mean fitness of the default decision, the evolved decision is used. The statistical hypothesis test used to this end is the Aspin–Welch–Satterthwaite (AWS) T–tests at a significance level of $\alpha = 0.05$. The AWS T–test is a statistical hypothesis test for the equality of means in which the equality of variances is not assumed [17].

6.3.3 Results

A population size of 100 was experimentally found to be adequate for solving the optimization problem in each time step. We set $t^{\text{end}} = 100$ and advanced time by a timestep of 1 every 5000 evaluations (i.e. every 50 generations). Since only one pattern was added to the dataset each timestep, and only a normal distribution is estimated from data, learning can be done very fast for this problem. Therefore learning was performed whenever time was advanced. The final results (i.e. the integral of the dynamic optimization function over $[0, 100]$) are shown in Figure 4 for different values of the prediction length t^{plen}. Indeed as expected and motivated earlier in the previous subsection, the best value for t^{plen} is not the maximum length of t^{end}, but a smaller length, even if the model is fully known.

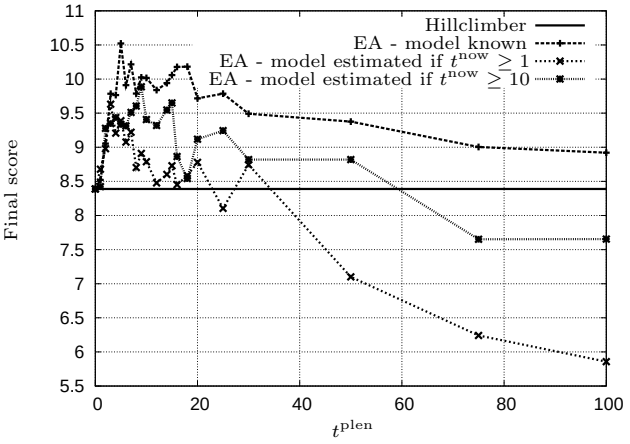

Figure 4: Final score (i.e. $\int_0^{t^{\text{end}}} \mathfrak{F}^{\text{dyn}}_{\boldsymbol{\gamma}^{\text{dyn}}(t)}(\boldsymbol{\zeta}(t))\,dt$) averaged over 100 runs on the dynamic pickup problem as a function of the prediction length.

The trajectory of the cumulative fitness for the best value and maximum value of t^{plen} are shown in Figure 5. This figure also reveals why the use of information about the future results in a better result in the end. All algorithms other than the hillclimber are willing to accept negative scores in a single turn if the prospect on future gains is larger. This of course happens if the truck moves more towards the origin as the density of the normal distribution is the highest there. The better strategy adopted by the system is thus to initially move towards the region close to the origin and never move too far away from it even if a profitable pickup can be made in a single turn by doing so.

Finally, it is again interesting to note that postponing the use of learning until a higher reliability is obtained leads to better results, indicating again the importance of reliable predictions in the proposed approach.

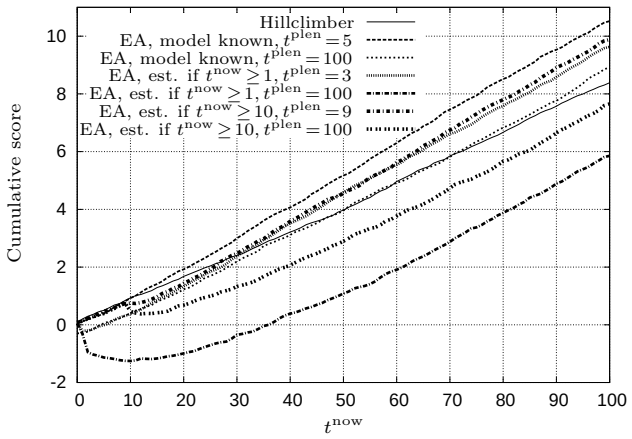

Figure 5: Cumulative score (i.e. $\mathfrak{H}^{\text{dyn}}(t^{\text{now}}, \boldsymbol{\zeta})$) averaged over 100 runs on the dynamic pickup problem.

7. DISCUSSION AND CONCLUSIONS

In this paper we have highlighted a specific source of difficulty in online dynamic optimization problems. We have labeled the difficulty time–linkage. In the worst case time–linkage can lead to time–deception. In that case any optimization algorithm is mislead and finds suboptimal results unless future implications of current decisions are taken into account. To tackle problems exhibiting this type of problem difficulty, we have proposed a framework that learns to predict the future and optimizes not only the current situation but also future predicted situations. We have proposed and used two new benchmark problems, but a larger suite of problems containing time–linkage is called for and should become a standard in dynamic optimization research.

In our experiments, we have fixed the future prediction timespan as well as the history data timespan. An interesting question is whether the timespans required to prevent deception can be measured during optimization. This calls for techniques for time–linkage identification in a similar sense as gene–linkage identification techniques are required in standard GAs to prevent deception as a result of dependencies between a problem's variables [12, 26].

Another important and related issue is how quickly the reliability of prediction degrades into the future. Even if we know how far into the future we must predict, it is hardly of any use to use these predictions if they are unreliable. The prediction reliability is influenced mostly by the difficulty of the function to predict (i.e. relatively steady or heavily fluctuating) and by the availability of data.

Ultimately, the expansion of dynamic EAs to process time–linkage information should be integrated with current state–of–the–art dynamic EAs that are capable of tackling other important problem difficulties that arise in dynamic optimization such as the overtaking of the optima by other local optima as time goes by. An EA that is capable of efficiently tackling both sources of problem difficulty is likely to be robust and well–suited to be used in practice and hence to be tested in real–world scenario's. To that end however, a further expansion that makes the approach well–suited for the multi–objective case is also likely to be crucial.

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