Reference Frame and Scale Invariant Real-parameter Genetic and Differential Evolution Algorithms

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We consider 'invariance' in the context of optimization algorithms. 'Scale invariance' implies algorithm performance that is independent of uniform scaling of all the design variables. 'Frame invariance' in turn implies algorithm performance that is independent of frame translation and rotation.

The notion of scale and frame invariance is not new to the field of evolutionary computation. In particular, rotation invariance has been investigated in previous research. Salomon [2] showed that most evolutionary algorithms are rotationally variant since the decomposability of test functions are tacitly assumed. In addition, Ono and coworkers [1] introduced the unimodal normal distribution crossover (UNDX) operator for the real-parameter genetic algortihm (RPGA) specifically to deal with non-decomposable test functions or functions that have epitasis amongst the design variables.

Frame indifference (or objectivity), is well known in classical mechanics, where physical laws dictate that this principle must hold. However, no corresponding law in optimization theory requires that an optimization algorithm must be frame invariant. It might therefore seem that frame indifference is merely an aesthetic requirement. If a particular optimization algorithm is frame dependent, it follows that there exists a specific choice of reference frame in which the problem can be solved easier (i.e. requiring less iterations) or better (i.e. achieving a lower cost function value) as compared to some other reference frame. In general, the performance difference in different reference frames cannot be quantified, and depends amongst others on the optimization problem and algorithm specifics. Since *a priori* knowledge of the optimal reference frame for a particular problem is seldom available, additional burden is placed on the analyst, which now has to consider solving the problem in a number of reference frames. (An exception being the specialized class of separable functions.) If the algorithm's frame dependency is severe, the analyst requires some external procedure to take the algorithm's frame dependency into account. A conceptual procedure is to recast the optimization problem to simultaneously solve for the reference frame and solution, but this renders the problem ill posed.

A frame invariant algorithm requires less *a priori* knowledge (or tacit assumptions) of the optimization problem, as compared to a frame variant algorithm. This implies that a frame invariant algorithm will have either inferior or superior performance as compared to a frame variant algorithm, depending on the validity of the assumed information.

In this study we discuss alternative strategies to unimodal normal distribution crossover (UNDX) in order to obtain rotationally invariant RPGAs. In addition, we also discuss rotationally variant mutation operators and show how they can be made rotationally invariant in order to obtain scale and frame invariant optimization algorithms.

In addition we also show that the differential evolution (DE) algorithm is rotationally variant. We obtain a rotationally invariant DE algorithm by identifying the operations that are variant and modifying them to be invariant.

Retaining diversity in the RPGA and DE algorithms is essential when making them invariant. Many invariant operators lack the diversity requirement and the challenge is therefore to introduce operators that are both diverse and invariant. We accomplish this by merely perturbing the direction cosines of chosen difference vectors. This introduces an additional parameter. We also discuss alternative strategies to obtain invariant operators that are diverse.

We quantify the (in)variance and performance of the RPGA and DE algorithms using popular test suite problems. The test problems are evaluated in both unrotated and arbitrarily rotated, translated and scaled reference frames.

The aim of this study is not to develop an algorithm that is competitive compared to well understood and researched algorithms, but to rather show that frame invariant algorithms can easily be constructed. We are however pleasantly surprised by the positive results obtained with our first attempt. We aim to investigate the invariant algorithms further in order to better understand the effect of the newly introduced parameters.

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1. REFERENCES

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