Binary Particle Swarm Optimization–A Forma Analysis Approach

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1. INTRODUCTION

PSO was originally designed as a numerical optimization technique based on swarm intelligence [1]. This paper aims to generalize PSO in a rigorous manner with *forma analysis* [2]. Thus, binary PSO operators can be formally derived in a principled manner.

Forma analysis is a formal but practical method that allows the problem representation and its operators to be structured in a formal manner by using *equivalence relations*. Since equivalence relations/classes have the ability to capture the properties of solutions, concrete operators can thus be mathematically derived by manipulating these equivalence relations in a formal way.

Our approach (as illustrated in Figure 1) can be explained as, *given an operator template, any suitable description of the considered problem domain gives rise to a concrete operator, whose behaviors and performance are related to the assumption we make to describe the search space.*

2. PSO OPERATOR TEMPLATE

The generalization of PSO includes the generalization of both position and velocity. Particularly, the velocity can be generalized as the "directed" step size for perturbation, which is effectively the distance between the previous two positions. Thus, the operator template can be formally defined as:

$$dis_\Psi(X_{i(t+1)}, X_{it}) = w * dis_\Psi(X_{it}, X_{i(t-1)}) + c_1 r_1 * dis_\Psi(Pb, X_{it}) + c_2 r_2 * dis_\Psi(Gb, X_{it})\}. \quad (1)$$

where Pb and Gb represent the personal best and the global best correspondingly, Ψ represents the basis, while i and t represent the index and the time step correspondingly.

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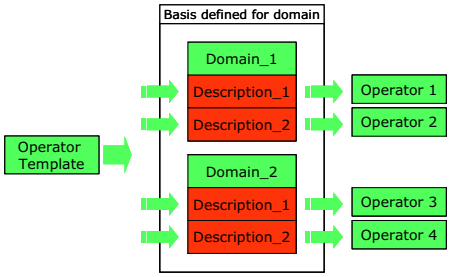

Figure 1: Illustration of the methodology based on forma analysis.

3. BINARY PSO SCHEMES

To represent the binary-string solution following the forma analysis formalism, equivalence relation can be defined for each binary position while the equivalence classes induced by the corresponding equivalence relation ψ_d are $\{\xi_d^0, \xi_d^1\}$.

By considering different assumptions of the basis on *dimensionality* and *direction*, we obtain 3 binary PSO schemes:

1. BPSO1–when all decision variables are regarded as a single dimension, the *accumulation distance* (e.g., k) is applied to the current solution by flipping the values for k randomly selected variables.

2. BPSO2–when each decision variable is regarded as a single dimension and the direction of difference is ignored, the perturbation of the current solution is jointly decided by the weighted sum of inertia weight and the greediness for personal/global best for each position.

3. BPSO3–when each decision variable is regarded as a single dimension and the direction of difference is retained, the perturbation of the current solution along with its direction is jointly decided by the weighted sum of inertia weight and the greediness towards personal/global best for each position.

In all these cases, the accumulation of distances can be interpreted in terms of probability of applying a change.

4. REFERENCES

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[2] N. Radcliffe. The algebra of genetic algorithms. *Annals of Maths and Artificial Intelligence*, 10:339–384, 1994.