Evolving Neural Networks

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Why Neuroevolution?





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- Neural nets powerful in many statistical domains
 - E.g. control, pattern recognition, prediction, decision making
 - Where no good theory of the domain exists
- · Good supervised training algorithms exist
 - Learn a nonlinear function that matches the examples
- What if correct outputs are not known?

Sequential Decision Tasks



- POMDP: Sequence of decisions creates a sequence of states
- No targets: Performance evaluated after several decisions
- Many important real-world domains:
 - Robot/vehicle/traffic control
 - Computer/manufacturing/process optimization
 - Game playing

Forming Decision Strategies



- Traditionally designed by hand
 - Too complex: Hard to anticipate all scenarios
 - Too inflexible: Cannot adapt on-line
- Need to discover through exploration
 - Based on sparse reinforcement
 - Associate actions with outcomes

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Neuroevolution (NE) Reinforcement Learning



- NE = constructing neural networks with evolutionary algorithms
- Direct nonlinear mapping from sensors to actions
- Large/continuous states and actions easy
 - Generalization in neural networks
- Hidden states disambiguated through memory
 - Recurrency in neural networks⁶⁹

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How well does it work?



- Difficult RL benchmark: Non-Markov Pole Balancing
- NE 3 orders of magnitude faster than standard RL
- NE can solve harder problems



- Powerful method for sequential decision tasks^{21,46,81}
 - Optimizing existing tasks
 - Discovering novel solutions
 - Making new applications possible
- Also may be useful in supervised tasks^{41,51}
 - Especially when network topology important
- Unique model of biological adaptation and development^{47,56,75}

Outline

- Basic neuroevolution techniques
- Advanced techniques
 - E.g. combining learning and evolution
- Extensions to applications
- Application examples
 - Control, Robotics, Artificial Life, Games

Neuroevolution Decision Strategies

- Input variables describe the state
- Output variables describe actions
- Network between input and output
 - Hidden nodes
 - Weighted connections
- Execution:
 - Numerical activation of input
 - Nonlinear weighted sums
- Performs a nonlinear mapping
 - Memory in recurrent connections
- Connection weights and structure evolved

Output units



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Conventional Neuroevolution (CNE)



- Evolving connection weights in a population of networks ^{41,57,81,82}
- Chromosomes are strings of weights (bits or real)
 - E.g. 10010110101010101111001
 - Usually fully connected, fixed topology
 - Initially random

Conventional Neuroevolution (2)



- Each NN evaluated in the task
 - Good NN reproduce through crossover, mutation
 - Bad thrown away
 - Over time, NNs evolve that solve the task
- Natural mapping between genotype and phenotype
- GA and NN are a good match!

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Problems with CNE



- Evolution converges the population (as usual with EAs)
 - Diversity is lost; progress stagnates
- Competing conventions
 - Different, incompatible encodings for the same solution
- Too many parameters to be optimized simultaneously
 - Thousands of weight values at once

Advanced NE 1: Evolving Neurons



- Evolving individual neurons to cooperate in networks^{1,45,51}
- E.g. Enforced Sub-Populations (ESP²¹)
 - Each (hidden) neuron in a separate subpopulation
 - Fully connected; weights of each neuron evolved
 - Populations learn compatible subtasks



Evolving Neurons with ESP



- Evolution encourages diversity automatically
 - Good networks require different kinds of neurons
- Evolution discourages competing conventions
 - Neurons optimized for compatible roles
- Large search space divided into subtasks
 - Optimize compatible neurons

Advanced NE 2: Evolutionary Strategies



- Evolving complete networks with ES (CMA-ES³⁰)
- Small populations, no crossover
- Instead, intelligent mutations
 - Adapt covariance matrix of mutation distribution
 - Take into account correlations between weights
- Smaller space, less convergence, fewer conventions ¹⁶

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Advanced NE 3: Evolving Topologies



- Optimizing connection weights and network topology ^{3,20,83}
- E.g. Neuroevolution of Augmenting Topologies (NEAT^{61,64})
- Based on Complexification
- Of networks:
 - Mutations to add nodes and connections
- Of behavior:
 - Elaborates on earlier behaviors

How Can Crossover be Implemented?

• Problem: Structures do not match



• Solution: Utilize historical markings



How can Innovation Survive?

• Problem: Innovations have initially low fitness



- Solution: Speciate the population
 - Innovations have time to optimize
 - Mitigates competing conventions
 - Promotes diversity

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How Can We Search in Large Spaces?

• Need to optimize not just weights but also topologies



- Solution: Start with minimal structure and complexify
 - Hidden nodes, connections, input features⁷⁹



Advanced NE 4: Indirect Encodings

E E P E starting E network cell

- Instructions for constructing the network evolved
 - Instead of specifying each unit and connection ^{3,39,59,83}
- E.g. Cellular Encoding (CE²⁷)
- Grammar tree describes construction
 - Sequential and parallel cell division
 - Changing thresholds, weights
 - A "developmental" process that results in a network 21

Properties of Indirect Encodings

- Smaller search space
- Avoids competing conventions
- Describes classes of networks efficiently
- Modularity, reuse of structures
 - Recurrency symbol in CE: $XOR \rightarrow parity$
 - Useful for evolving morphology
- Not all that powerful (yet)
- Promising current work
 - More general L-systems; developmental codings; embryogeny⁶⁵
 - Spatial coding ¹³ (D'Ambrosio GECCO'07)

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 Genetic Regulatory Networks⁵² (Reisinger GECCO'07)

How Do the NE Methods Compare?



Two poles, no velocities, 2 different setups:

- Advanced methods better than CNE
- Advanced methods are still improving
- Indirect encodings future work
- DEMO

Further NE Techniques

- Incremental evolution^{23,71,82}
- Utilizing population culture^{5,37}
- Evolving ensembles of NNs^{33,50,77}
- Evolving neural modules⁵³
- Evolving transfer functions and learning rules^{8,55,68}
- Combining learning and evolution



Combining Learning and Evolution

- Good learning algorithms exist for NN
 - Why not use them as well?
- Evolution provides structure and initial weights
- Fine tune the weights by learning
- Lamarckian evolution is possible
 - Coding weight changes back to chromosome
- Difficult to make it work
 - Diversity reduced; progress stagnates

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Baldwin Effect

- Learning can guide Darwinian evolution^{4,28}
 - Makes fitness evaluations more accurate
- With learning, more likely to find the optimum if close

Without learning

Genotype

- Can select between good and bad individuals better
 - Lamarckian not necessary
- How can we implement it?
 - How to obtain training targets?

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Targets from a Related Task



- Learning in a related task is sufficient
- E.g. foraging for food in a microworld⁴⁷
 - Network sees the state, outputs motor commands
 - Trained with backprop to predict the next input
 - Training emphasizes useful hidden-layer representations
 - Allows more accurate evaluations

Evolving the Targets



- Evolve extra outputs to provide targets
- E.g. in the foraging task⁴⁹
 - Motor outputs and targets with separate hidden layers
 - Motor weights trained with backprop, targets evolved
 - Targets do not correspond to optimal performance: Direct system towards useful learning experiences

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Targets from the Population



- Train new offspring to imitate parents/champion³⁷
 - Trained in population "culture"
- Local search around good individuals
 - Limited training: 8-20 backprop iterations
- Becomes part of the evaluation
 - Individuals evolve to anticipate training
 - Perform poorly at birth, well after training
- Evolution discovers optimal starting points for learning!

Targets from Q-Learning



- E.g. NEAT+Q⁷⁸: Evolve network to represent value function
 - Input is the state, outputs are Q-values of actions
- Form targets according to Q-learning equations
 - Compare successive Q-values, use backprop to train
- Improves evolution of a value function
 - Faster than NEAT alone, better than Q-learning
- Utilize both evolution and on-line learning

No Targets: Unsupervised Learning



- Hebbian adaptation during performance^{17,62}
- E.g. handwritten character recognition⁷⁴
 - Evolution determines the starting point
 - Competitive learning finishes the design
- Starting points are poor recognizers
 - Only bias learning away from local minima
- Synergetic effect: Evolution utilizes learning
- Future work: Constructing developmental systems

Extending NE to Applications

- Control
- Robotics
- Artificial life
- Gaming

Issues:

- Evolving composite decision makers⁷⁷
- Evolving teams of agents 6,63,84
- Utilizing coevolution ^{54,66}
- Real-time neuroevolution⁶³
- Combining human knowledge with evolution 7,15,86

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Applications to Control



- Pole-balancing benchmark
 - Originates from the 1960s
 - Original 1-pole version too easy
 - Several extensions: acrobat, jointed, 2-pole, particle chasing⁵⁰
- Good surrogate for other control tasks
 - Vehicles and other physical devices
 - Process control⁷²



Controlling a Finless Rocket

Task: Stabilize a finless version of the Interorbital Systems RSX-2 sounding rocket²⁴

- Scientific measurements in the upper atmosphere
- 4 liquid-fueled engines with variable thrust
- Without fins will fly much higher for same amount of fuel

Active Rocket Guidance



- Used on large scale launch vehicles (Saturn, Titan)
- Typically based on classical linear feedback control
- High level of domain knowledge required
- Expensive, heavy

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Rocket Stability



Simulation Environment: JSBSim



- General rocket simulator
- Models complex interaction between airframe, propulsion, aerodynamics, and atmosphere
- Used by IOS in testing their rocket designs
- Accurate geometric model of the RSX-2

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Rocket Guidance Network









Results: Control Policy

Driving and Collision Warning



- · Goal: evolve a collision warning system
 - Looking over the driver's shoulder
 - Adapting to drivers and conditions
 - Collaboration with Toyota³²

The RARS Domain



- RARS: Robot Auto Racing Simulator
 - Internet racing community
 - Hand-designed cars and drivers
 - First step towards real traffic

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Evolving Good Drivers



- Evolving to drive fast without crashing (off road, obstacles)
- An interesting challenge of its own⁷⁰
- Discovers optimal driving strategies (e.g. how to take curves)
- Works from range-finder & radar inputs
- Works from raw visual inputs
- DEMO

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Evolving Warnings



- Evolving to estimate probability of crash
- Predicts based on subtle cues (e.g. skidding off the road)
- Compensates for disabled drivers
- Human drivers learn to drive with it!
- DEMO

Applications to Robotics

- Controlling a robot arm ⁴⁴
 - Compensates for an inop motor
- Robot walking^{29,58}
 - Various physical platforms
- Mobile robots ^{11,16,48,60}
 - Transfers from simulation to physical robots
 - Evolution possible on physical robots



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Robotic Soccer

- E.g. robocup soccer "Keepaway" task⁷⁷
- Three keepers, one (algorithmic) taker
- Includes many behaviors:

Get-Open, Intercept, Evaluate-Pass, Pass...

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Direct Evolution



- Mapping sensors directly to actions
 - Difficult to separate behaviors
 - Ineffective combinations evolve
- DEMO

Cooperative Coevolution



- Evolve multiple actions
 - Each one in a separate network
 - Decision tree to decide on actions
 - Or evolve a decision network

Cooperative Coevolution (2)



- Networks learn individual tasks
- Learn to anticipate other tasks
 - Lining up for a pass
- Cooperative coevolution of composite behavior
- DEMO

Applications to Artificial Life



Gaining insight into neural structure
 E.g. evolving a command neuron^{2,31,56}

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- Emergence of behaviors
 Signaling, herding, hunting...^{75,76,85}
- Future challenges
 - Emergence of language
 - Emergence of community behavior

Competitive Coevolution



- Evolution requires an opponent to beat
- Such opponents are not always available
- Co-evolve two populations to outdo each other
- How to maintain an arms race?⁴⁰

Competitive Coevolution with NEAT



- Complexification elaborates instead of alters
 - Adding more complexity to existing behaviors
- Can establish a coevolutionary arms race
 - Two populations continually outdo each other
 - Absolute progress, not just tricks

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Robot Duel Domain



- Two Khepera-like robots forage, pursue, evade ⁶⁶
 - Collect food to gain energy
 - Win by crashing to a weaker robot

Early Strategies



- Crash when higher energy
- Collect food by accident
- DEMO





- Collect food to gain energy
- Avoid moving to lose energy
- Standoff: Difficult to predict outcome
- DEMO

A Sophisticated Strategy



- "Fake" a move up, force away from last piece
- Win by making a dash to last piece
- $\bullet \ \ \ Complexification \to arms \ race$
- DEMO

Applications to Games





- Good research platform ³⁸
 - Controlled domains, clear performance, safe
 - Economically important; training games possible
- Board games: beyond limits of search
 - Evaluation functions in checkers, chess^{9,18,19}
 - Filtering information in go, othello^{42,67}
 - Opponent modeling in poker³⁴ (Lockett, Chen GECCO'07)

Discovering Novel Strategies in Othello



- Players take turns placing pieces
- Each move must flank opponent's piece
- Surrounded pieces are flipped
- Player with most pieces wins

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Strategies in Othello



- Positional
 - Number of pieces and their positions
 - Typical novice strategy
- Mobility
 - Number of available moves: force a bad move
 - Much more powerful, but counterintuitive
 - Discovered in 1970's in Japan

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Evolving Against a Random Player



- Network sees the board, suggests moves by ranking⁴³
- Networks maximize piece counts throughout the game
- A positional strategy emerges
- Achieved 97% winning percentage





- lago's positional strategy destroyed networks at first
- Evolution turned low piece count into an advantage
- Mobility strategy emerged!
- Achieved 70% winning percentage

Example game





- Black's positions strong, but mobility weak
- White (the network) moves to f2
- Black's available moves b2, g2, and g7 each will surrender a corner
- The network wins by forcing a bad move

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Discovering Novel Strategies



- Neuroevolution discovered a strategy novel to us
- "Evolution works by tinkering"
 - So does neuroevolution
 - Initial disadvantage turns into novel advantage

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Video Games



- Economically and socially important
- Adaptation an important future goal
 - More challenging, more fun games
 - Possible to use for training people
- How to make evolution run in real time?

Real-time NEAT



- A parallel, continuous version of NEAT⁶³
- $\bullet\,$ Individuals created and replaced every n ticks
- Parents selected probabilistically, weighted by fitness
- Long-term evolution equivalent to generational NEAT
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NERO: A Complex Game Platform



- Teams of agents trained to battle each other
 - Player trains agents through excercises
 - Agents evolve in real time
 - Agents and player collaborate in battle
- New genre: Learning is the game
- Challenging platform for reinforcement learning
 - Real time, open ended, requires discovery
- DEMO

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Utilizing Human Knowledge



- Given a problem, NE discovers a solution by exploring
 - Sometimes you already know (roughly) what works
 - Sometimes random initial behavior is not acceptable
- How can domain knowledge be utilized?
 - By incorporating rules 12,86
 - By learning from examples⁷

Incorporating Rules into NE



E.g. how to go around a wall in NERO

- Specify as a rule:
 - wall_ahead: move_forward, turn_right
 - wall_45deg_left, move_forward, turn_right_slightly
- Convert into a network with KBANN³⁵

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Incorporating Rules into NE (2)



- KBANN network added to NEAT networks
 - Treated as complexification
 - Continues to evolve
 - If advice is useful, it stays
- Initial behaviors, on-line advice
- Injecting human knowledge as rules
- DEMO

Lessons from NERO



- NEAT is a strong method for real-time adaptation
 - Complex team behaviors can be constructed
 - Novel strategies can be discovered
- Problem solving with human guidance
- Coevolutionary arms race
- NE makes a new genre of games possible!

(NERO details, download: http://nerogame.org)

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Numerous Other Applications

- Creating art, music¹⁰
- Theorem proving¹⁴
- Time-series prediction ³⁶
- Computer system optimization²²
- Manufacturing optimization²⁶
- Process control optimization 72,73
- Finding the top quark⁸⁰
- Etc.

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Evaluation of Applications



- Neuroevolution strengths
 - Can work very fast, even in real-time
 - Potential for arms race, discovery
 - Effective in continuous, non-Markov domains
- Requires many evaluations
 - Requires an interactive domain for feedback
 - Best when parallel evaluations possible
 - Works with a simulator & transfer to domain

Conclusion

- NE is a powerful technology for sequential decision tasks
 - Evolutionary computation and neural nets are a good match
 - Lends itself to many extensions
 - Powerful in applications
- · Easy to adapt to applications
 - Control, robotics, optimization
 - Artificial life, biology
 - Gaming: entertainment, training
- Lots of future work opportunities
 - Theory not well developed
 - Indirect encodings
 - Learning and evolution
 - Knowledge and interaction

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