Quantum Computing

A Tutorial at the
2007 Genetic and Evolutionary Computation Conference
(GECCO-2007)

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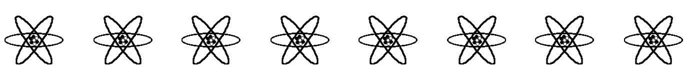
Overview

- What is quantum computation?
- Why might it be important?
- How does/might it work?
- Simulating a quantum computer.
- Some quantum algorithms.
- Evolution of new quantum algorithms.
- Sources for more information.

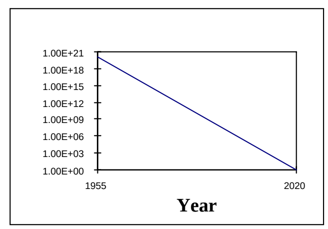
What is quantum computation?

Computation with coherent atomic-scale dynamics.



The behavior of a quantum computer is governed by the laws of quantum mechanics.

Why bother with quantum computation?



- Moore's Law: the amount of information storable on a given amount of silicon has roughly doubled every 18 months. We hit the quantum level 2010 ~ 2020.
- Quantum computation is more powerful than classical computation. More can be computed in less time—the complexity classes are different!

GECCO'07, July 7–11, 2007, London, England, United Kingdom.
ACM 978-1-59593-698-1/07/0007.

The power of quantum computation

- In quantum systems ***possibilities count***, even if they never happen!
- Each of exponentially many ***possibilities*** can be used to perform a part of a computation ***at the same time***.

Nobody understands quantum mechanics

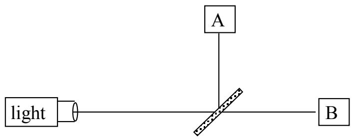
- "Anybody who is not shocked by quantum mechanics hasn't understood it." —Niels Bohr
- "No, you're not going to be able to understand it. ... You see, my physics students don't understand it either. That is because **I** don't understand it. Nobody does. ... The theory of quantum electrodynamics describes Nature as absurd from the point of view of common sense. And it agrees fully with experiment. So I hope you can accept Nature as She is—absurd." —Richard Feynman

Absurd but taken seriously

(not just quantum mechanics but also quantum computation)

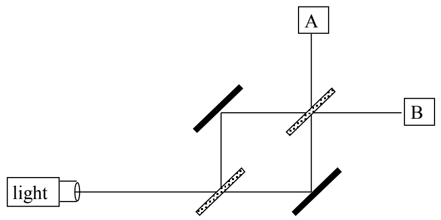
- Under active investigation by many of the top physics labs around the world (including CalTech, MIT, AT&T, Stanford, Los Alamos, UCLA, Oxford, l'Université de Montréal, University of Innsbruck, IBM Research...)
- In the mass media (including *The New York Times*, *The Economist, American Scientist, Scientific American*, ...)
- Here.

A beam splitter



Half of the photons leaving the light source arrive at detector A; the other half arrive at detector B.

An interferometer



- Equal path lengths, rigid mirrors.
- Only one photon in the apparatus at a time.
- All of the photons leaving the light source arrive at detector B. WHY?

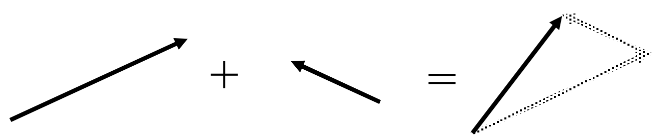
Possibilities count

- There is an "amplitude" for each possible path that a photon can take.
- The amplitudes can interfere constructively and destructively, even though each photon takes only one path.
- The amplitudes at detector A interfere destructively; those at detector B interfere constructively.

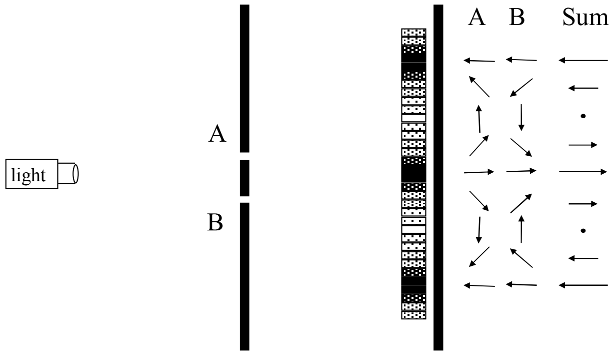
Calculating interference

- "You will have to brace yourselves for this—not because it is difficult to understand, but because it is absolutely ridiculous: All we do is draw little arrows on a piece of paper—that's all!" —Richard Feynman
- Arrows for each possibility.
- Arrows rotate; speed depends on frequency.
- Arrows flip 180˚ at mirrors, rotate 90˚ counter-clockwise when reflected from beam splitters.
- Add arrows and square the length of the result to determine the probability for any possibility.

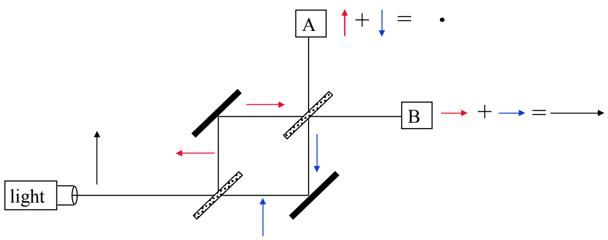
Adding arrows



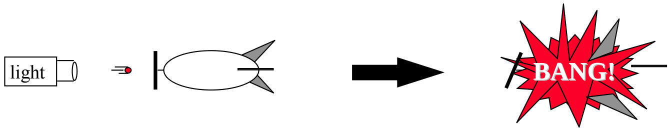
Double slit interference



Interference in the interferometer

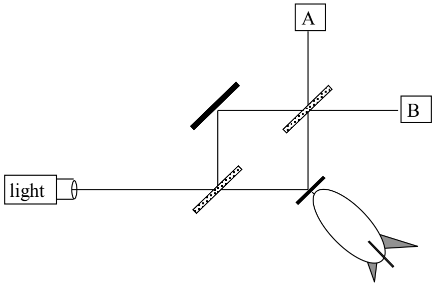


A photon-triggered bomb



- A mirror is mounted on a plunger on the bomb's nose.
- A single photon hitting the mirror depresses the plunger and explodes the bomb.
- Some plungers are stuck, producing duds.
- How can you find a good, unexploded bomb?

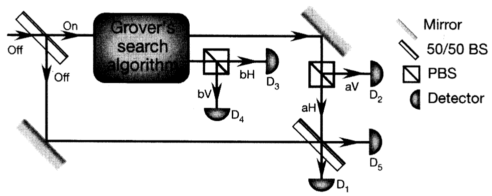
Elitzur-Vaidman bomb testing



- Possibilities count!
- Experimentally verified
- Can be enhanced to reduce or eliminate bomb loss
  [Kwiat, Weinfurter and Kasevich]

Counterfactual quantum computation

- Hosten et al. used optical counterfactual computation to conduct a search without running the search algorithm (*Nature* **439**, 23 Feb 2006).
- They also used a "chained Zeno effect"—a sequence of interferometers—to boost the inference probability to unity.



(Image scanned from the *Nature* article.)

Two interesting speedups

- Grover's quantum database search algorithm finds an item in an unsorted list of n items in $O(\sqrt{n})$ steps; classical algorithms require $O(n)$.
- Shor's quantum algorithm finds the prime factors of an n-digit number in time $O(n^3)$; the best known classical factoring algorithms require at least time $O(2^{n^{1/3} \log(n)^{2/3}})$.

Reminder: exponential savings is **very** good!

Factor a 5,000 digit number:

- Classical computer (1ns/instr, ~today's best alg)
  - » **over 5 trillion years** (the universe is ~ 10–16 billion years old).
- Quantum computer (1ns/instr, ~Shor's alg)
  - » just over 2 minutes

Quantum computing and the human brain

- Penrose's argument

  Brains do X (for X uncomputable)

  Classical computers can't do X

  ∴ Brains aren't classical computers

  - First premise is false for all proposed X. For example, brains don't have knowably sound procedures for mathematical proof.
  - Would imply brains more powerful than quantum computers; new physics.

Quantum consciousness?

- Relation to consciousness etc. is much discussed, unclear at best. (Bohm, Penrose, Hameroff, others)
- "[Penrose's] argument seemed to be that consciousness is a mystery and quantum gravity is another mystery so they must be related." (Hawking)

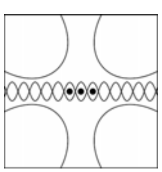
Quantum information theory

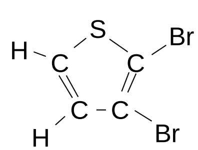
- Quantum cryptography: secure key distribution
- Quantum teleportation
- Quantum data compression
- Quantum error correction

Good introductions to these topics can be found in (Steane, 1998).

Physical implementation

- Ion traps
- Nuclear spins in NMR devices
- Optical systems
- So far: few qubits, impractical
- A lot of current research





Languages and notations

- Wave equations
- Wave diagrams
- Matrix mechanics
- Dirac's bra-ket notation ($\langle\phi|\psi\rangle$)
- Particle diagrams
- Amplitude diagrams
- Phasor diagrams
- QGAME programs

Qubits

- The smallest unit of information in a quantum computer is called a "qubit".
- A qubit may be in the "on" (1) state or in the "off" (0) state or in any superposition of the two!

State representation, 1 qubit

- The state of a qubit can be represented as:

  $\alpha_0|0\rangle + \alpha_1|1\rangle$

  α_0 and α_1 are complex numbers that specify the *probability amplitudes* of the corresponding states.
- $|\alpha_0|^2$ gives the probability that you will find the qubit in the "off" (0) state; $|\alpha_1|^2$ gives the probability that you will find the qubit in the "on" (1) state.

Entanglement

- Qubits in a multi-qubit system are not independent—they can become "entangled." (We'll see some examples.)
- To represent the state of n qubits one usually uses 2^n complex number amplitudes.

State representation, 2 qubits

- The state of a two-qubit system can be represented as:

  $\alpha_0|00\rangle + \alpha_1|01\rangle + \alpha_2|10\rangle + \alpha_3|11\rangle$

  $\Sigma\ |\alpha|^2 = 1$
- Measurement will always find the system in some (one) discrete state.

Measurement at the end of a computation

- $\Sigma|\alpha|^2$, for amplitudes of all states matching the output bit-pattern in question.
- This gives the probability that the particular output will be read upon measurement.
- Example:

  $0.316|00\rangle+0.447|01\rangle+0.548|10\rangle+0.632|11\rangle$

  The probability to read the rightmost bit as 0 is $|0.316|^2+|0.548|^2=0.4$

Partial measurement during a computation

- One-qubit measurement gates.
- Measurement changes the system.
- In simulation, branch computation for each possible measurement.

Classical computation in matrix form

A state transition in a 4-bit system:

$$\begin{bmatrix} 0&1&0&0&0&0&0&0&0&0&0&0&0&0&0&0 \\ 1&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0 \\ 0&0&1&0&0&0&0&0&0&0&0&0&0&0&0&0 \\ 0&0&0&1&0&0&0&0&0&0&0&0&0&0&0&0 \\ 0&0&0&0&1&0&0&0&0&0&0&0&0&0&0&0 \\ 0&0&0&0&0&1&0&0&0&0&0&0&0&0&0&0 \\ 0&0&0&0&0&0&1&0&0&0&0&0&0&0&0&0 \\ 0&0&0&0&0&0&0&1&0&0&0&0&0&0&0&0 \\ 0&0&0&0&0&0&0&0&1&0&0&0&0&0&0&0 \\ 0&0&0&0&0&0&0&0&0&1&0&0&0&0&0&0 \\ 0&0&0&0&0&0&0&0&0&0&1&0&0&0&0&0 \\ 0&0&0&0&0&0&0&0&0&0&0&1&0&0&0&0 \\ 0&0&0&0&0&0&0&0&0&0&0&0&1&0&0&0 \\ 0&0&0&0&0&0&0&0&0&0&0&0&0&1&0&0 \\ 0&0&0&0&0&0&0&0&0&0&0&0&0&0&1&0 \\ 0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&1 \end{bmatrix} * \begin{bmatrix} \alpha_0 \\ \alpha_1 \\ \alpha_2 \\ \alpha_3 \\ \alpha_4 \\ \alpha_5 \\ \alpha_6 \\ \alpha_7 \\ \alpha_8 \\ \alpha_9 \\ \alpha_{10} \\ \alpha_{11} \\ \alpha_{12} \\ \alpha_{13} \\ \alpha_{14} \\ \alpha_{15} \end{bmatrix}$$

A quantum NOT gate

$$\begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix}$$

Applied to a qubit:

$$\begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix} * \begin{pmatrix} \alpha_0 \\ \alpha_1 \end{pmatrix} = \begin{pmatrix} \alpha_1 \\ \alpha_0 \end{pmatrix}$$

$$\alpha_0|0\rangle + \alpha_1|1\rangle \longrightarrow \alpha_1|0\rangle + \alpha_0|1\rangle$$

Explicit matrix expansion

To expand gate matrix G for application to an n-qubit system:

– Create a 2^nx2^n matrix M.

– Let Q be the set of qubits to which the operator is being applied, and Q' be the set of the remaining qubits.

– $M_{ij} = 0$ if i and j differ in positions in Q'.

– Otherwise concatenate bits from i in positions Q to produce i^*, and bits from j to produce j^*. $M_{ij} = G_{i^*j^*}$.

Implicit matrix expansion

To apply gate matrix G to an n-qubit system:

– Let Q be the set of qubits to which the operator is being applied, and Q' be the set of the remaining qubits.

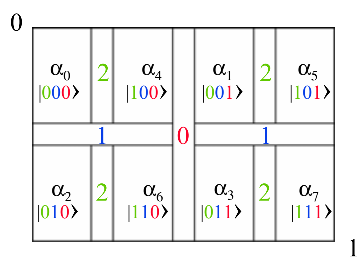
– For every combination C of 1 and 0 for qubits in Q':

» Extract the column A of amplitudes that results from holding C constant and varying all qubits in Q.

» $A' = G$ x A.

» Install A' in place of A in the array of amplitudes.

Amplitude diagrams



◆ Help to visualize amplitude distributions

◆ Scalable, hierarchical

◆ Can be shuffled to prioritize any qubits

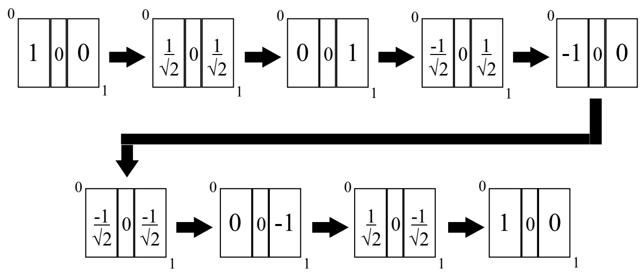
A square-root-of-NOT (SRN) gate

$$\begin{pmatrix} \frac{1}{\sqrt{2}} & -\frac{1}{\sqrt{2}} \\ \frac{1}{\sqrt{2}} & \frac{1}{\sqrt{2}} \end{pmatrix}$$

◆ Applied once to a classical state, this ~randomizes the value of the qubit.

◆ Applied twice in a row, this is ~equivalent to NOT: $\begin{bmatrix} \frac{1}{\sqrt{2}} & -\frac{1}{\sqrt{2}} \\ \frac{1}{\sqrt{2}} & \frac{1}{\sqrt{2}} \end{bmatrix} * \begin{bmatrix} \frac{1}{\sqrt{2}} & -\frac{1}{\sqrt{2}} \\ \frac{1}{\sqrt{2}} & \frac{1}{\sqrt{2}} \end{bmatrix} = \begin{bmatrix} 0 & -1 \\ 1 & 0 \end{bmatrix}$

SRN amplitude diagrams



Other quantum gates

- Rotation ($U\theta$): $\begin{bmatrix} \cos(\theta) & \sin(\theta) \\ -\sin(\theta) & \cos(\theta) \end{bmatrix}$
- Hadamard (H): $\frac{1}{\sqrt{2}}\begin{bmatrix} 1 & 1 \\ 1 & -1 \end{bmatrix}$
- Controlled NOT ($CNOT$): $\begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 1 \\ 0 & 0 & 1 & 0 \end{bmatrix}$

There are many small "complete" sets of gates [Barenco et al.].

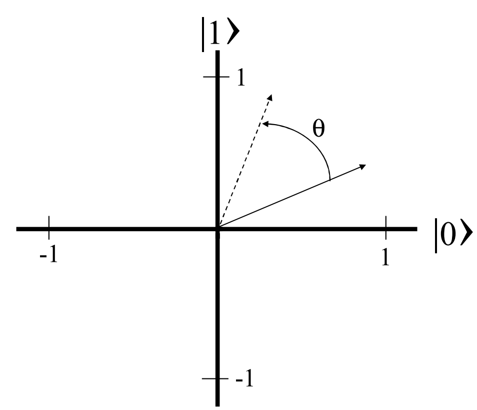
More quantum gates

- Conditional phase: $\begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & e^{i\alpha} \end{bmatrix}$
- $U2$: $\begin{bmatrix} e^{-i\phi} & 0 \\ 0 & e^{i\phi} \end{bmatrix} \times \begin{bmatrix} \cos(\theta) & \sin(-\theta) \\ \sin(\theta) & \cos(\theta) \end{bmatrix} \times \begin{bmatrix} e^{-i\psi} & 0 \\ 0 & e^{i\psi} \end{bmatrix} \times \begin{bmatrix} e^{i\alpha} & 0 \\ 0 & e^{i\alpha} \end{bmatrix}$

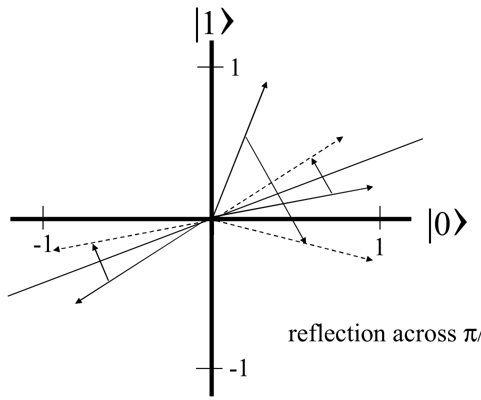
All gates must be unitary: $U^{\dagger}U = UU^{\dagger} = I$,

where $U^{\dagger}$ is the Hermitean adjoint of U, obtained by taking the complex conjugate of each element of U and then transposing the matrix.

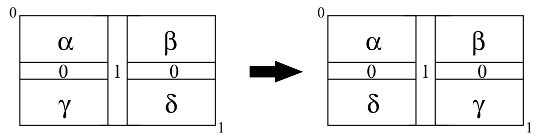
Rotation polar plot for real vectors



Hadamard polar plot for real vectors



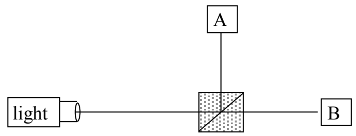
CNOT amplitude diagrams



CNOT(0 [control], 1 [target])

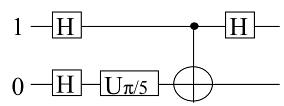
Polarizing beam-splitter CNOT gate

[Cerf, Adami, and Kwiat]



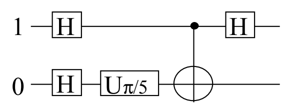
- Two qubits encoded in one photon, one in momentum (direction) and one in polarization.
- Polarization controls change in momentum.
- Cannot be scaled up directly, but demonstrates an implementation of a 2-qubit gate.

Gate array diagrams

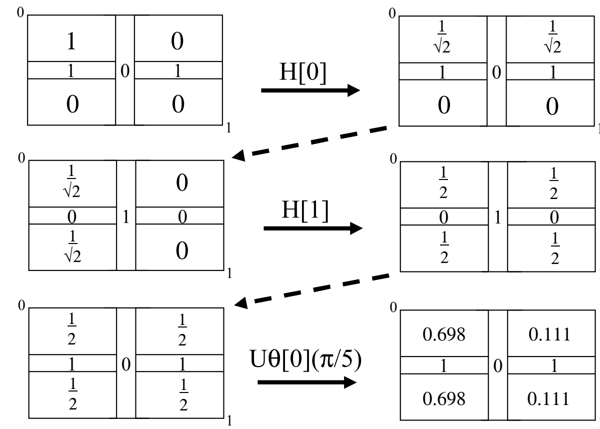


Example execution trace

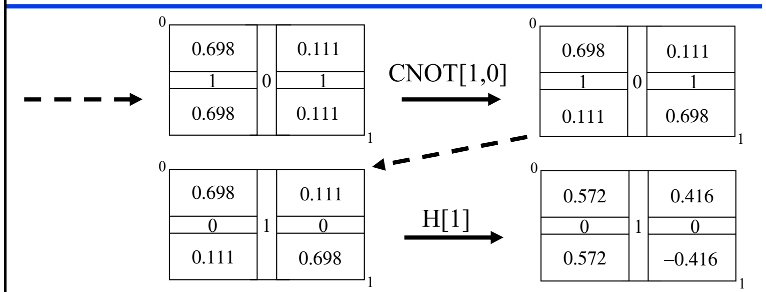
```
Hadamard qubit:0
Hadamard qubit:1
U-theta qubit:0 theta:pi/5
Controlled-not control:1 target:0
Hadamard qubit:1
```



Trace, cont.



Trace, cont.



state	probability
|00›	0.33
|01›	0.33
|10›	0.17
|11›	0.17

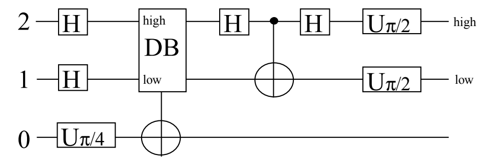
The database search problem

- Given an unsorted database containing *n* items but only one “marked” item, find the address of the marked item with a minimal number of database calls.
- Lov Grover’s algorithm uses $O(\sqrt{n})$ calls in general, and only one call for a 4-item database.

Oracle problems

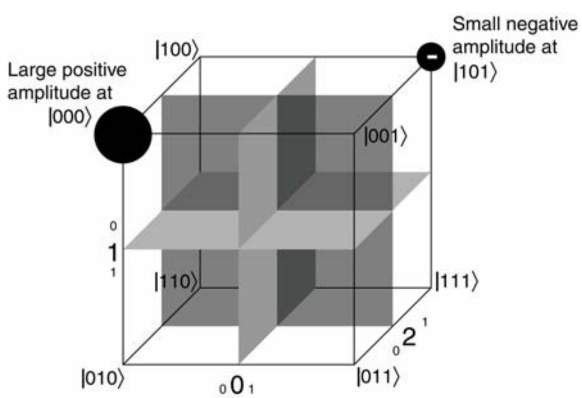
- The database search problem is an example of an "oracle problem."
- We are given a "black box" or "oracle" function (in this case the database access function) and asked to find out if it has some particular property.
- Many other known quantum algorithms are for oracle problems.
- Often the oracle is "hard" to implement, so complexity is figured from the number of oracle calls.

Grover's algorithm for a 4-item database

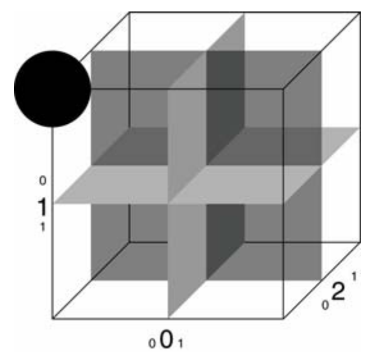


- Start in the state |000›.
- Read answer from qubits 2 and 1.

Cube diagram for a 3-qubit system

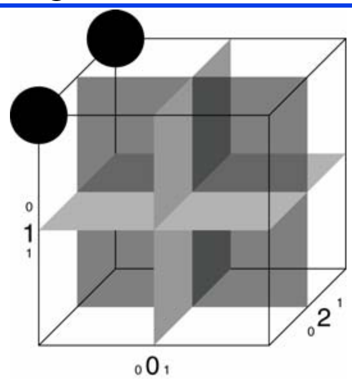


(0) Grover's algorithm, item at 0,0



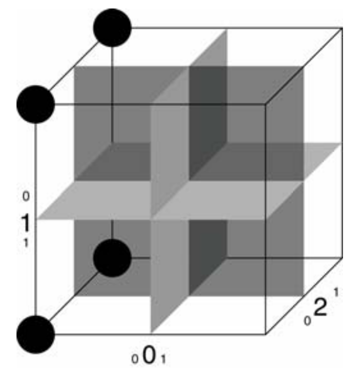
Initial State, |000>

(1) Grover's algorithm, item at 0,0



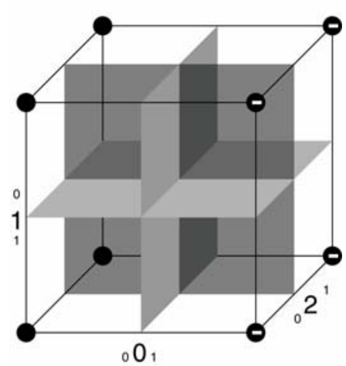
After Hadamard[2]

(2) Grover's algorithm, item at 0,0



After Hadamard[1]

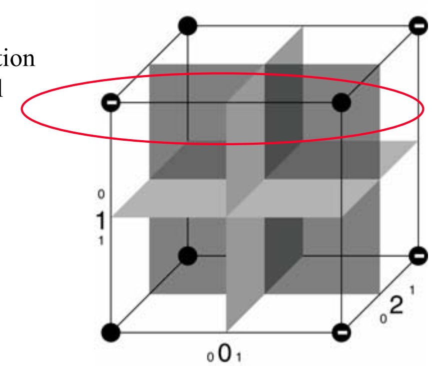
(3) Grover's algorithm, item at 0,0



After Uθ[0](π/4)

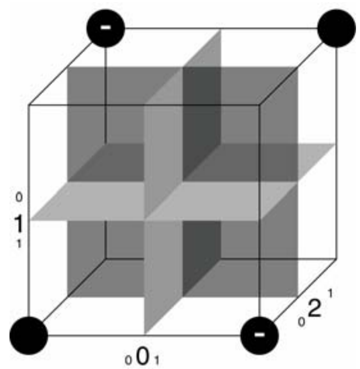
(4) Grover's algorithm, item at 0,0

Note position
of DB call
effect.



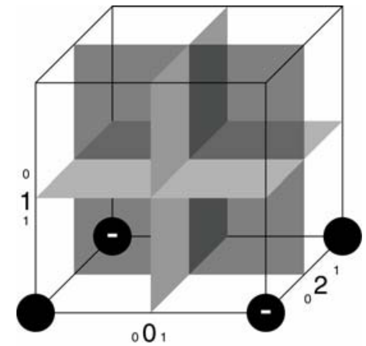
After Database Call [in: 2,1; out:0]

(5) Grover's algorithm, item at 0,0



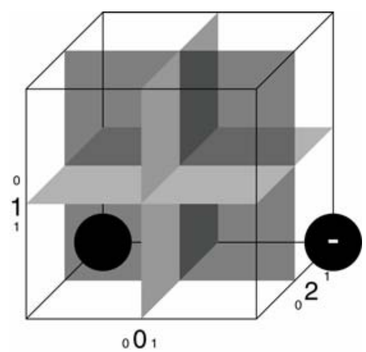
After Hadamard[2]

(6) Grover's algorithm, item at 0,0



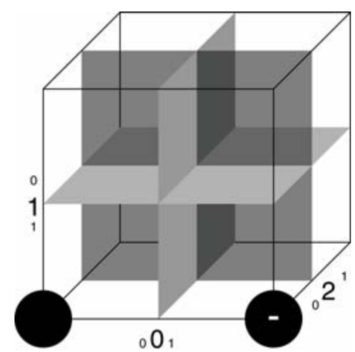
After CNOT [control: 2; target: 1]

(7) Grover's algorithm, item at 0,0



After Hadamard[2]

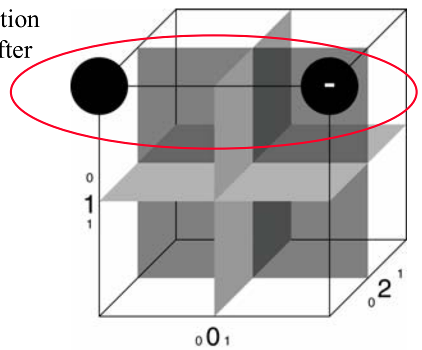
(8) Grover's algorithm, item at 0,0



After Uθ[2](π/2)

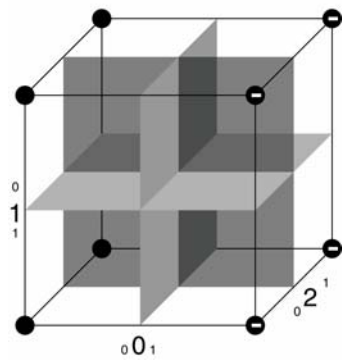
(9) Grover's algorithm, item at 0,0

Note relation to state after DB call.



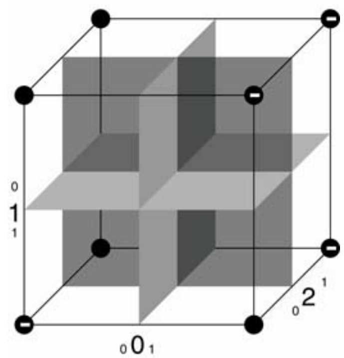
After Uθ[1](π/2), Read output from qubits 2 (high) and 1(low)

(3) Grover's algorithm, item at 0,1



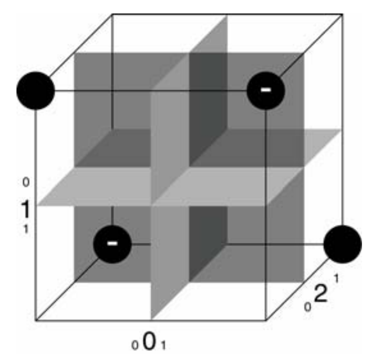
After Uθ[0](π/4)

(4) Grover's algorithm, item at 0,1



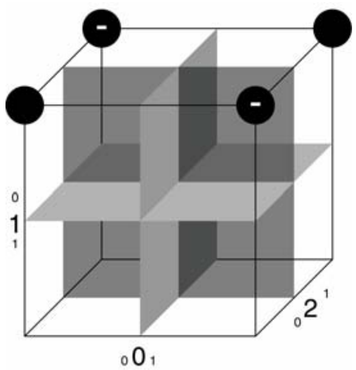
After Database Call [in: 2,1; out:0]

(5) Grover's algorithm, item at 0,1



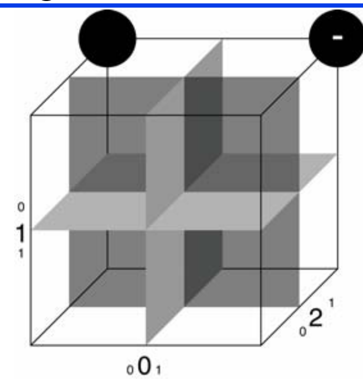
After Hadamard[2]

(6) Grover's algorithm, item at 0,1



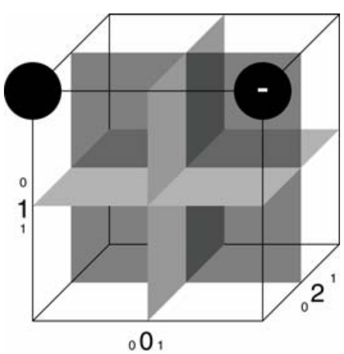
After CNOT [control: 2; target: 1]

(7) Grover's algorithm, item at 0,1



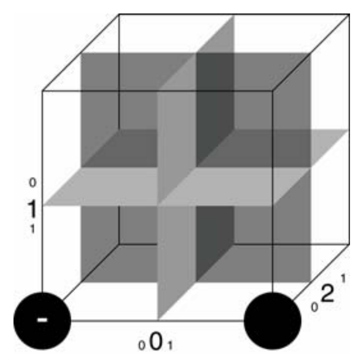
After Hadamard[2]

(8) Grover's algorithm, item at 0,1



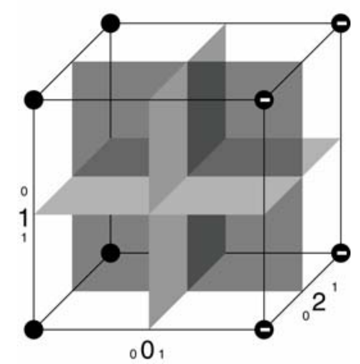
After Uθ[2](π/2)

(9) Grover's algorithm, item at 0,1



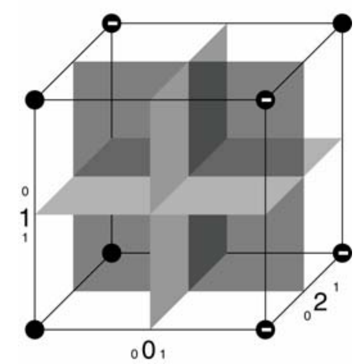
After Uθ[1](π/2), Read output from qubits 2 (high) and 1(low)

(3) Grover's algorithm, item at 1,0



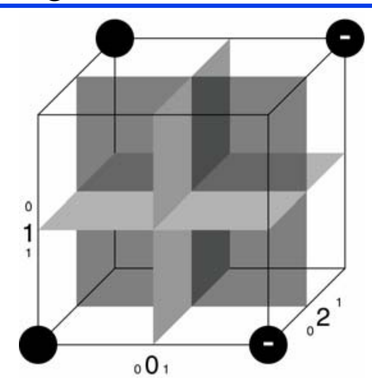
After Uθ[0](π/4)

(4) Grover's algorithm, item at 1,0



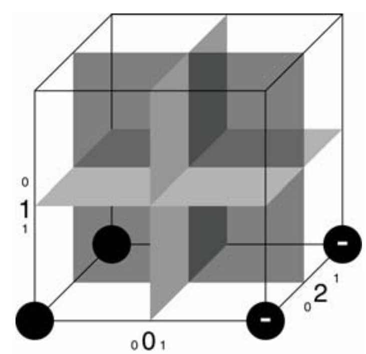
After Database Call [in: 2,1; out:0]

(5) Grover's algorithm, item at 1,0



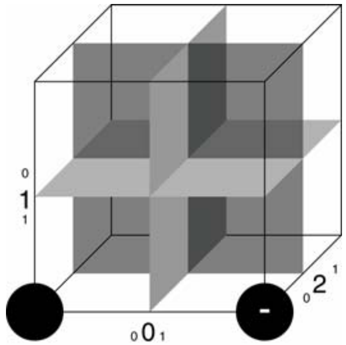
After Hadamard[2]

(6) Grover's algorithm, item at 1,0



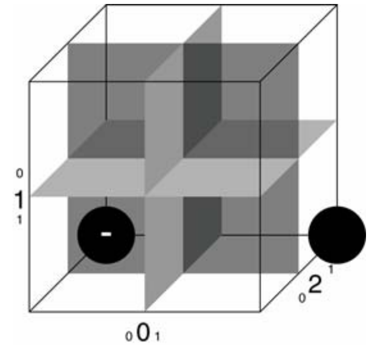
After CNOT [control: 2; target: 1]

(7) Grover's algorithm, item at 1,0



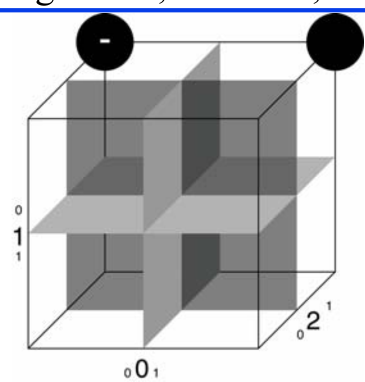
After Hadamard[2]

(8) Grover's algorithm, item at 1,0



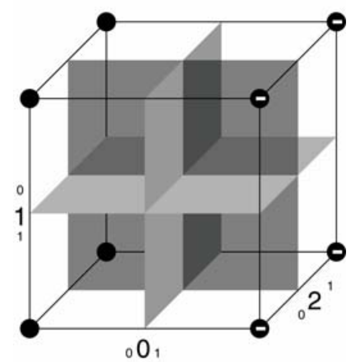
After Uθ[2](π/2)

(9) Grover's algorithm, item at 1,0



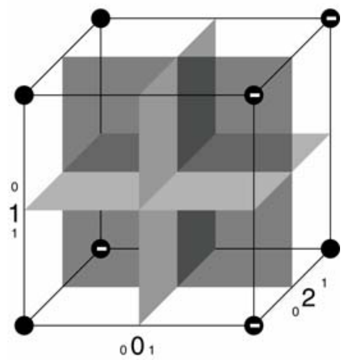
After Uθ[1](π/2), Read output from qubits 2 (high) and 1(low)

(3) Grover's algorithm, item at 1,1



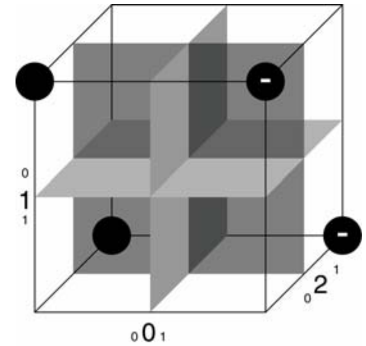
After Uθ[0](π/4)

(4) Grover's algorithm, item at 1,1



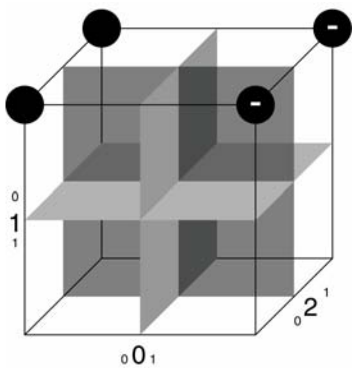
After Database Call [in: 2,1; out:0]

(5) Grover's algorithm, item at 1,1



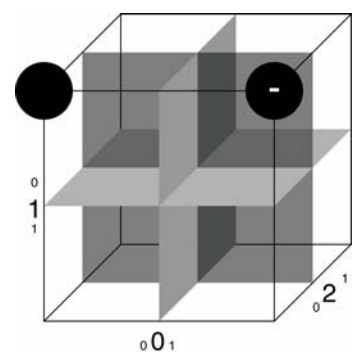
After Hadamard[2]

(6) Grover's algorithm, item at 1,1



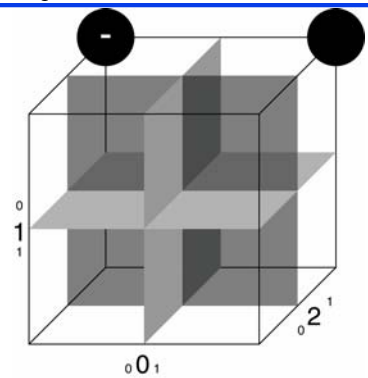
After CNOT [control: 2; target: 1]

(7) Grover's algorithm, item at 1,1



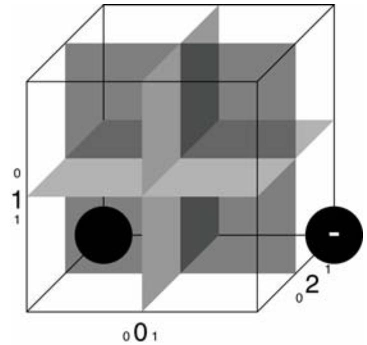
After Hadamard[2]

(8) Grover's algorithm, item at 1,1



After Uθ[2](π/2)

(9) Grover's algorithm, item at 1,1

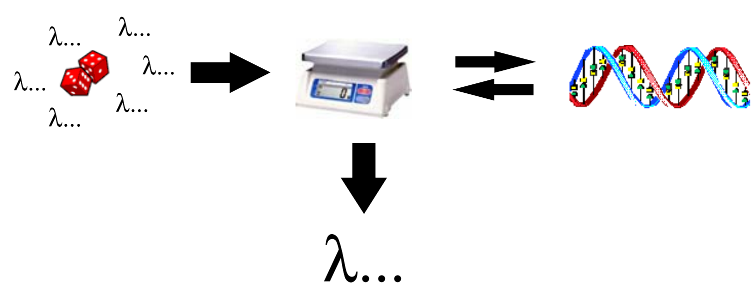


After Uθ[1](π/2), Read output from qubits 2 (high) and 1(low)

Shor's algorithm

- hybrid algorithm to factor numbers
- quantum component helps to find the period r of a sequence a_1, a_2, ... a_i, ... , given an oracle function that maps i to a_i
- skeleton of the algorithm:
  - create a superposition of all oracle inputs
  - call the oracle function
  - apply a quantum Fourier transform to the input qubits
  - read the input qubits to obtain a random multiple of $1/r$
  - repeat a small number of times to infer r

Genetic Programming (GP)



GP for quantum computation

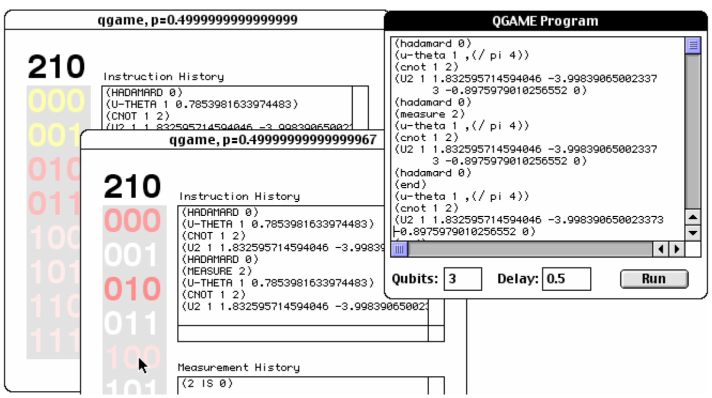
- Evolve:
  - gate arrays
  - programs that produce gate arrays
  - hybrid classical/quantum algorithms
  - input states or parameters
- Genome representation:
  - QGAME program
  - program (in any language) that generates a QGAME program
  - array of numbers

Fitness

- Assessing the composite matrix
  - the trouble with oracles
- Assessing the results of simulation runs
- Criteria:
  - Error
  - Hits
  - Oracle calls
  - Number of gates

QGAME Quantum Gate and Measurement Emulator

http://hampshire.edu/lspector/qgame.html



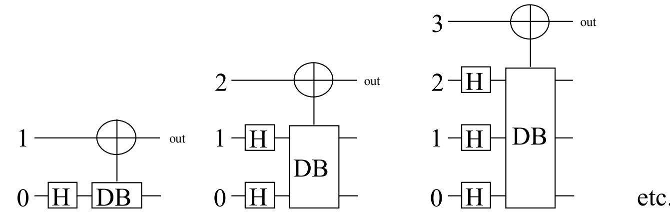
Primitives; gate-array-producing programs

- Gates: H, U_θ, $CNOT$, $ORACLE$, ...
- Qubit indices
- Gate parameters (angles)
- Arithmetic operators
- Constants indicating problem size (num-qubits, num-input-qubits, num-output-qubits)
- Iteration structures, recursion, data structures, …

The scaling majority-on problem

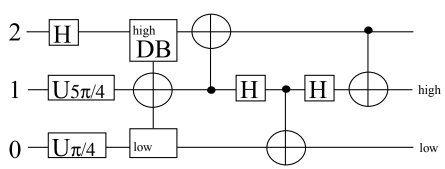
- Does the oracle answer “1” for a majority of inputs?
- Seek program that produces a gate array for any oracle size.

Evolved scaling majority-on gate arrays

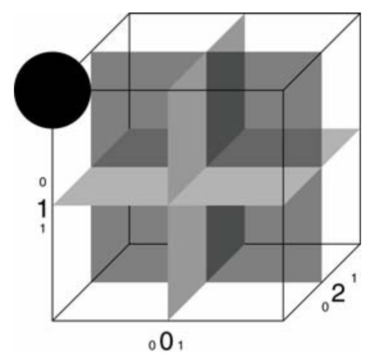


Not better than classical.

Evolved database search gate array

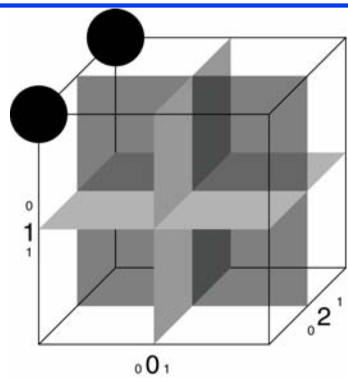


(0) Evolved quantum database algorithm, item at 0,0



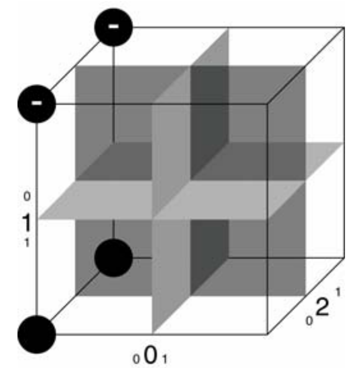
Initial State, |000>

(1) Evolved quantum database algorithm, item at 0,0



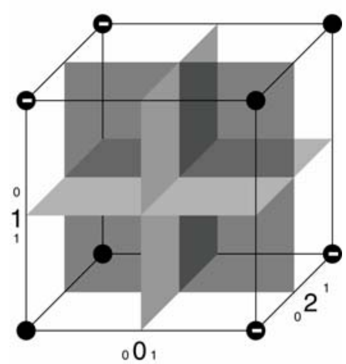
After Hadamard [2]

(2) Evolved quantum database algorithm, item at 0,0



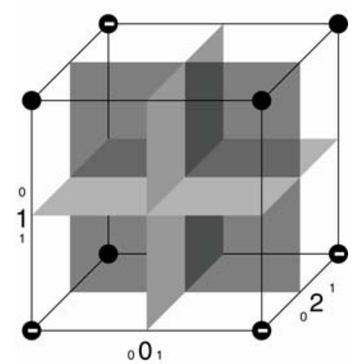
After Uθ [1] (5π/4)

(3) Evolved quantum database algorithm, item at 0,0



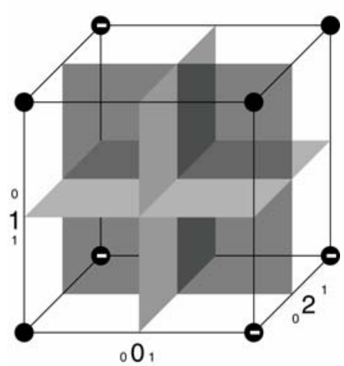
After Uθ [0] (π/4)

(4) Evolved quantum database algorithm, item at 0,0



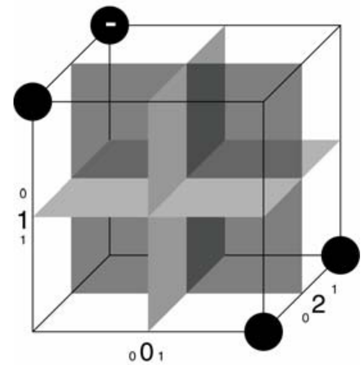
After DB [in:2,0; out:1](item in 0,0)

(5) Evolved quantum database algorithm, item at 0,0



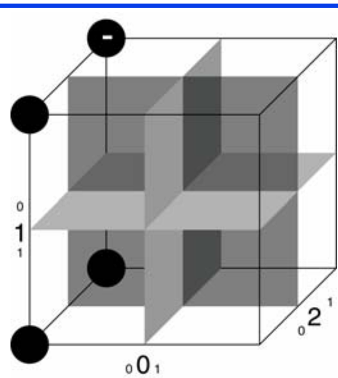
After CNOT [control: 1, target: 2]

(6) Evolved quantum database algorithm, item at 0,0



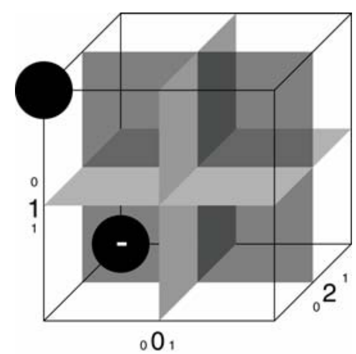
After Hadamard [1]

(7) Evolved quantum database algorithm, item at 0,0



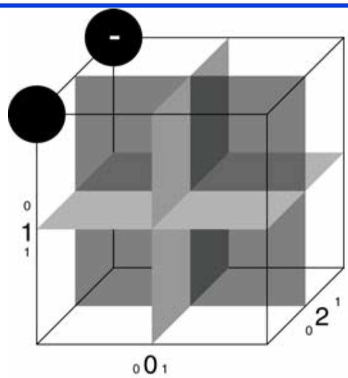
After CNOT [control: 1, target: 0]

(8) Evolved quantum database algorithm, item at 0,0



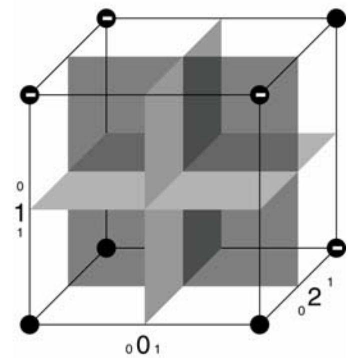
After Hadamard [1]

(9) Evolved quantum database algorithm, item at 0,0



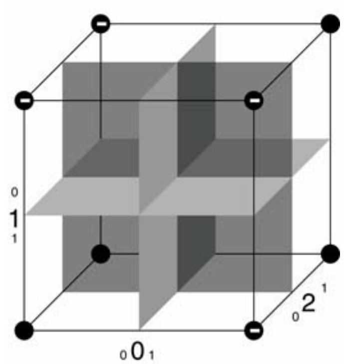
After CNOT [control: 2, target: 1]
Read output from qubits 1 (high) and 0(low)

(4) Evolved quantum database algorithm, item at 0,1



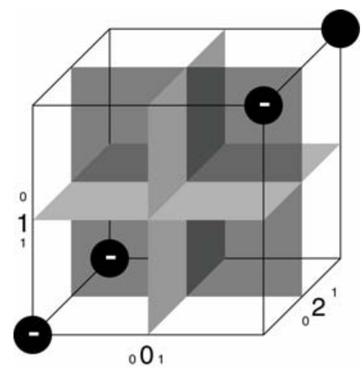
After DB [in:2,0; out:1](item in 0,1)

(5) Evolved quantum database algorithm, item at 0,1



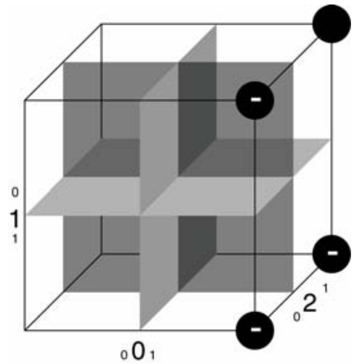
After CNOT [control: 1, target: 2]

(6) Evolved quantum database algorithm, item at 0,1



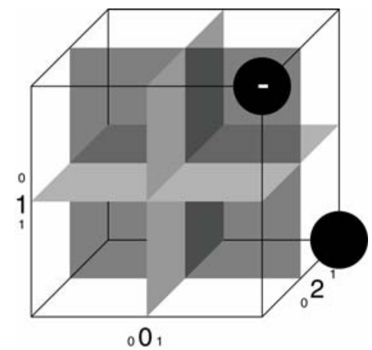
After Hadamard [1]

(7) Evolved quantum database algorithm, item at 0,1



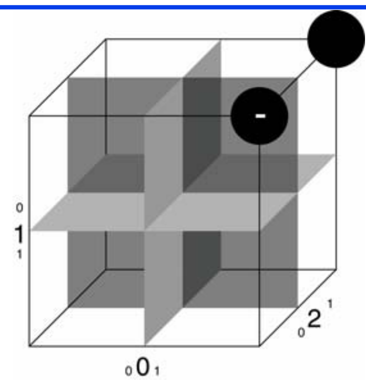
After CNOT [control: 1, target: 0]

(8) Evolved quantum database algorithm, item at 0,1



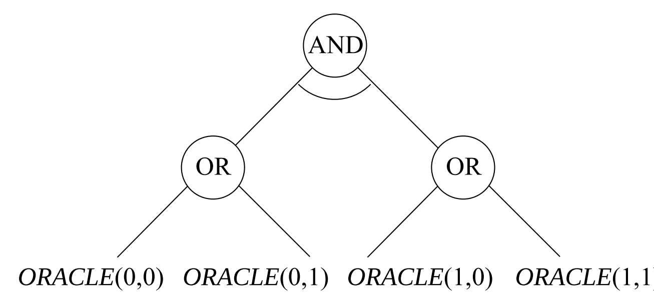
After Hadamard [1]

(9) Evolved quantum database algorithm, item at 0,1

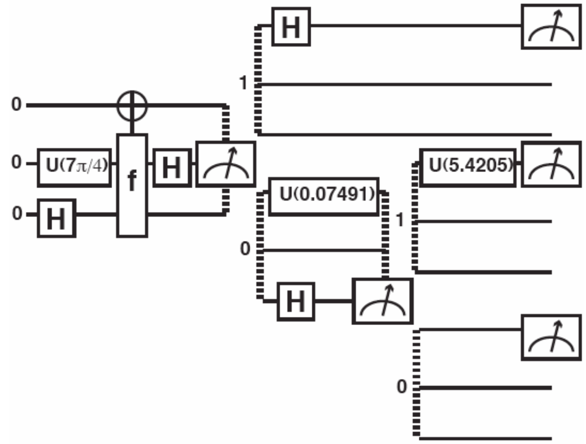


After CNOT [control: 2, target: 1]
Read output from qubits 1 (high) and 0(low)

The and-or tree problem



Evolved and-or gate array



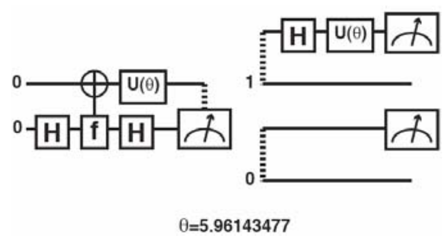
Error/complexity measures

- *Las Vegas* ≡ always correct, but may answer "don't know" with some probability
- *Monte Carlo* ≡ may err, with some probability
- p^e_{max} ≡ worst case probability of error
- q^e_{max} ≡ worst case expected queries
- *Exact* ≡ $p^e_{max} = 0$

Complexity of 2-bit AND/OR

- Classical Las Vegas: $q^e_{max}=3$
  - derived from [Saks and Wigderson 1986]
- Classical Monte Carlo: for $q^e_{max}=1$, $p^e_{max}\geq 1/3$
  - derived from [Santha 1991]
- Evolved Quantum Monte Carlo: $p^e_{max} = 0.28732$

Derived better-than-classical OR

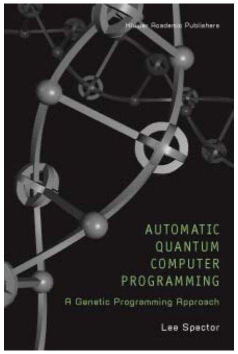


- Classical Monte Carlo: for $q^e_{max}=1$, $p^e_{max}\geq 1/6$ [Jozsa 1991, Beals 1998]
- Evolved algorithm $q^e_{max}=1$, $p^e_{max}=1/10$

GP/QC research directions

- Application to additional problems with incompletely understood quantum complexity
- Exploration of communication capacity of quantum gates
- Evolution of hybrid quantum/classical algorithms.
- Evolution guided by ease of physical implementation.
- QC applications in AI
  - general AI search?
  - and-or trees and Prolog: quantum logic machine?
  - Bayesian networks?
- Genetic programming *on* quantum computers.

Book



Automatic Quantum Computer Programming: A Genetic Programming Approach

Lee Spector. 2004. New York: Springer Science+Business Media. (Originally published by Kluwer Academic Publishers. Paperback edition 2007.)

http://hampshire.edu/lspector/aqcp/

Sources: selected articles

- A. Steane, 1998. "Quantum Computing," *Reports on Progress in Physics*, vol. 61, pp. 117-173. http://xxx.lanl.gov/abs/quant-ph/9708022
- P. Shor, 1998. "Quantum Computing," *Documenta Mathematica*, vol. Extra Volume ICM, pp. 467–486. http://east.camel.math.ca/EMIS/journals/DMJDMV/xvol-icm/00/Shor.MAN.ps.gz
- J. Preskill, 1997. "Quantum Computing: Pro and Con," Tech. Rep. CALT-68-2113, California Institute of Technology. http://xxx.lanl.gov/abs/quant-ph/9705032
- A. Barenco, C. H. Bennett, R. Cleve, D. P. DiVincenzo, N. Margolus, P. Shor, T. Sleator, J. Smolin, H. Weinfurter, 1995. "Elementary Gates for Quantum Computation," submitted to *Physical Review A*. http://xxx.lanl.gov/abs/quant-ph/9503016
- N.J. Cerf, C. Adami, P.G. Kwiat, 1998. "Optical Simulation of Quantum Logic," *Phys. Rev. A* 57, 1477. http://xxx.lanl.gov/abs/quant-ph/9706022
- L. Spector and H.J. Bernstein. 2003. "Communication Capacities of Some Quantum Gates, Discovered in Part through Genetic Programming," in *Proc. of the Sixth Intl. Conf. on Quantum Communication, Measurement, and Computing*, edited by J.H. Shapiro and O. Hirota. Princeton, NJ: Rinton Press, Inc. pp. 500–503. http://hampshire.edu/lspector/pubs/spector-QCMC-prepress.pdf
- H. Barnum, H.J. Bernstein, and L. Spector. 2000. Quantum circuits for OR and AND of ORs. Journal of Physics A: Mathematical and General, Vol. 33 No. 45 (17 November 2000), pp. 8047–8057. http://hampshire.edu/lspector/pubs/jpa.pdf
- L. Spector, H. Barnum, H.J. Bernstein, N. Swamy, 1999. "Quantum Computing Applications of Genetic Programming," in *Advances in Genetic Programming 3*, pp. 135–160, MIT Press.
- L. Spector, H. Barnum, H.J. Bernstein, N. Swamy, 1999. "Finding a Better-Than-Classical Quantum AND/OR Algorithm Using Genetic Programming," in *Proc. 1999 Congress on Evolutionary Computation*, IEEE Press.
- L. Spector, H. Barnum, H.J. Bernstein, 1998. "Genetic Programming for Quantum Computers," in *Genetic Programming 1998: Proceedings of the Third Annual Conference*, pp. 365–374, Morgan Kaufmann.

Sources: selected books

- *Automatic Quantum Computer Programming: A Genetic Programming Approach.* By Lee Spector. Kluwer Academic Publishers, 2004, and Springer Science+Business Media, 2007.
- *Quantum Computation and Quantum Information.* By Michael A. Nielsen and Isaac L. Chuang. Cambridge University Press. 2000.
- *Schrödinger's Machines: The Quantum Technology Reshaping Everyday Life.* By Gerard J. Milburn. W.H. Freeman and Company. 1997.
- *Explorations in Quantum Computing.* By Colin P. Williams and Scott H. Clearwater. Springer-Verlag/Telos. 1997.
- *The Fabric of Reality.* By David Deutsch. Penguin Books. 1997.
- *The Large, the Small and the Human Mind.* By Roger Penrose, with Abner Shimony, Nancy Cartwright, and Stephen Hawking. Cambridge University Press. 1997.
- *QED: The Strange Theory of Light and Matter.* By Richard P. Feynman. Princeton University Press. 1985.

Sources: selected WWW sites

- Oxford's Center for Quantum Computation: http://www.qubit.org/
- Stanford-Berkeley-MIT-IBM NMR Quantum Computation Project: http://squint.stanford.edu/
- Quantum Information and Computation (Caltech - MIT - USC): http://theory.caltech.edu/~quic/index.html
- Quantum Computation at ISI/USC: http://www.isi.edu/acal/quantum/quantum_intro.html
- Los Alamos National Laboratory quantum physics e-print archive: http://xxx.lanl.gov/form/quant-ph
- John Preskill's Physics 229 course web page (many good links): http://www.theory.caltech.edu/people/preskill/ph229/
- Samuel L. Braunstein's on-line tutorial: http://www.sees.bangor.ac.uk/~schmuel/comp/comp.html
- NIST Ion Storage Group: http://www.bldrdoc.gov/timefreq/ion/index.htm
- QGAME, Quantum Gate And Measurement Emulator: http://hampshire.edu/lspector/qgame.html