Efficiency Optimization of a Multi-Pump Booster System

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ABSTRACT

This paper discusses a way to optimize the speed settings for a multi-pump system such that it can operate with highest efficiency. A short description of an actual multi-pump system is given and the mathematical formulas for single pump and multi-pump systems are presented. Then a set of objectives are formulated which, when used in a MATLAB toolbox implementation of NSGA-II, describe the most efficient distribution of speeds amongst the pumps in the system. The system is tested for a number of pressure references with good results.

Categories and Subject Descriptors

J.2 [**Computer Applications**]: Physical Sciences and Engineering; J.6 [**Computer Applications**]: Computer-aided engineering—*Computer-aided design*

General Terms

Performance, Design

Keywords

Pump optimization, Parallel pumps, Multi-objective optimization, NSGA-II

1. INTRODUCTION

Pumps are some of the most used electrical components worldwide [2]. Whenever a liquid needs to be moved from one place to another, and it is not possible to just let gravity handle the transfer, a pump is used. Since the use of pumps is so widespread it is important to keep the efficiency in mind when choosing a pump for a specific application. If not, then a lot of energy will quickly go to waste.

The first step in optimizing efficiency of pumps can be made by varying the speeds at which they run. A pump that runs at a constant speed without regard to the load of the system connected to it has to be chosen such that it can handle the worst case pressure/flow scenario. That also means that for non-worst case running scenarios it will be running too fast and will be wasting energy since the excess energy only will raise the pressure of the system.

Just varying the speed of a pump is not sufficient to optimize the operation of a pump since the efficiency of these variable speed pumps varies with speed. Depending on the type of pump, a variable speed pump is most efficient at a specific speed and certain pressure/flow conditions. In fact, the type name of pumps usually include specifications of the optimal flow conditions.

When pumping liquids it is commonplace to control the increase in pressure of the liquid. Depending on the load of the system, the speed of the pump is varied to achieve the desired pressure. However, for some applications there can occur large variations in the flow of the liquid, including water supply for high-rise buildings and water supply for fire extinguishing. In such cases a pump will often operate in ranges where the efficiency of the pump is significantly lower than the optimally rated conditions. It might be more efficient to use several smaller pumps instead of using a single large pump. The problem then is to determine the best combination of speeds for the small pumps such that the overall efficiency of the pumping unit is as high as possible.

This paper investigates how the most efficient combination of speeds for a multi-pump booster system can be found using a Multi-Objective Evolutionary Algorithm (MOEA). Before reaching that goal it is necessary to first take a look at such a multi-pump booster system. In Section 2 a description of the system is given, both physically as well as a mathematical model. The mathematical model in Section 2.2 also explains how the equations for a single pump can be combined to describe a system with several pumps in parallel. In that section, the coefficients relating the mathematical model to the physical system are also given.

In Section 2.3, the objective functions describing the efficiency of the system is given and in Section 3 the MOEA used to solve the problem (NSGA-II)[1] is described along with implementation details. The obtained results are shown in Section 4 followed by a conclusion in Section 5.

2. MULTI-PUMP BOOSTER SYSTEM

This section will give an introduction to the physical multi-pump booster system which is used as case study in this paper. Following that a mathematical description of a single and multiple pumps in parallel are presented along with the corresponding coefficients for the system under consideration.

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2.1 Physical System

The physical system consists of 3 CRE5-8 pumps mounted in parallel between an inlet pipe and an outlet pipe. The speed of each pump can be controlled independently of the others and the power consumed by each pump can be monitored. Further, measurements of the total flow in the system, Q_{tot}, as well as the pressure difference from intake pipe to outlet pipe, Δp, are available. It is thus possible to perform measurements on each individual pump as well as the entire system such that the coefficients of the mathematical model, which is described next, can be calculated.

2.2 Mathematical Model

The mathematical model of the system is going to be used as the basic element in the optimization process since it will be performed offline and should result in a set of optimal speeds for the multi-pump system under different circumstances. First, a mathematical model of one of the pumps will be given and then extended to the case for multiple pumps in parallel.

Before starting with the mathematical model of the pump it is important that the technical terms used in connection with pumps are clear to the reader. The main difference is how to express the pressure generated by the pumps. Instead of using pressure difference Δp to indicate the increase in pressure from inlet to outlet it is common to use the term head given by

$$H = \frac{\Delta p}{g\rho} \ , \tag{1}$$

where g is the gravity constant and ρ is the density of the liquid being pumped. The reason for this change is that the head represents the height to which a non-specified liquid can be pumped. The equations thus become independent on the liquid in the system and the thermal properties of said liquid as well.

2.2.1 Single Pump

The mathematical model for a pump is a static model showing the relationship between the head and flow for different system loads. Due to various losses the relationship between head and flow are normally given by a second-, third-, or fourth-order polynomial with regard to the flow, Q. In this particular case a second order polynomial will be used resulting in the following pump curve that varies according to different pump speeds, ω.

$$H(\omega) = a(\omega)Q(\omega)^2 + b(\omega)Q(\omega) + c(\omega) \ . \tag{2}$$

The coefficients $a(\omega)$, $b(\omega)$, and $c(\omega)$ vary according to the speed ω but can be approximated using the affinity laws [2]

$$Q(\omega_B) = \frac{\omega_B}{\omega_A} Q(\omega_A) \tag{3}$$

$$H(\omega_B) = \left(\frac{\omega_B}{\omega_A}\right)^2 H(\omega_A) \ . \tag{4}$$

These are good approximations to the actual change in pump curve when the speed changes, provided the system curve remains unchanged. The approximated pump curve then becomes

$$H(\omega) = a_0 Q(\omega)^2 + b_0 \frac{\omega}{\omega_0} Q(\omega) + c_0 \left(\frac{\omega}{\omega_0}\right)^2 \ , \tag{5}$$

where a_0, b_0, and c_0 are the coefficients for the pump curve at the base speed ω_0. Thus, if a_0, b_0, c_0, and ω_0 are known then an approximate pump curve can be calculated for any speed ω.

The amount of power delivered from the pump to the liquid is called the hydraulic power and is given by

$$P_{hyd}(\omega) = H(\omega) g \rho Q(\omega) \ . \tag{6}$$

For hydraulic power the affinity law yields

$$P_{hyd}(\omega_B) = \left(\frac{\omega_B}{\omega_A}\right)^3 P_{hyd}(\omega_A) \ , \tag{7}$$

which indicates that the power consumption of the pump depends on the speed to the third power. So, when measuring the electrical power consumption of the pump, which includes losses in the pump and motor as well as the hydraulic power, it should be expected that the power depends on the speed to the third power. The mathematical model of the pump should thus include a third order polynomial representing the power consumption of the pump which can be based on measurements of the actual pump under consideration. The expression for the power consumption can thus be expressed as

$$P_{el}(\omega) = a_{el}\omega^3 + b_{el}\omega^2 + c_{el}\omega + d_{el} \ . \tag{8}$$

The efficiency of a pump is given by the relationship between the amount of power delivered to the liquid and the total power consumed,

$$\eta = \frac{P_{hyd}(\omega)}{P_{el}(\omega)} \ . \tag{9}$$

With the mathematical model of a single pump defined it is now time to look at the mathematical model for multiple pumps in parallel.

2.2.2 Multiple Parallel Pumps

When several pump are connected in parallel the total head is identical to the head generated by each individual pump, whereas the total flow in the system is the sum of the flows for each pump. So, if a system consists of n pumps the total flow of the system will be

$$Q_{tot}(\vec{\omega}) = \sum_{i=1}^{n} Q_i(\omega_i) \ , \tag{10}$$

where $\vec{\omega}$ is a vector containing the speeds ω_i of each pump i, and the head will be

$$H_{tot}(\vec{\omega}) = H_1(\omega_1) = \ldots = H_n(\omega_n) \ . \tag{11}$$

A graphical representation of the pump curve resulting from putting several pumps in parallel is given in Figure 1. Note that the pump-curves are based on actual pumps in the multi-pump system and have not been illustrated with head but with pressure when pumping water.

The effect of adding additional pumps to a system in parallel does not mean that only the flow will increase. If the system load is unchanged an addition of a pump will result in an increase of both flow and pressure. This is because the flow of liquid contributed from the extra pump will take up some of the space in the pipes of the system. The original pump(s) will thus see the addition of the extra pump as a

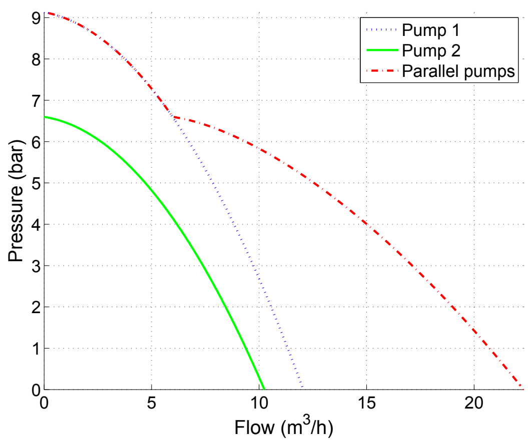

Figure 1: Pump curve for individual pumps and multiple pumps in parallel.

narrowing of the pipes, which corresponds to a higher system load. The contribution from the original pump(s) will then have a higher pressure and less flow. The extra pump will, however, also be operating at this increased head and still yield additional flow. This is illustrated in Figure 2.

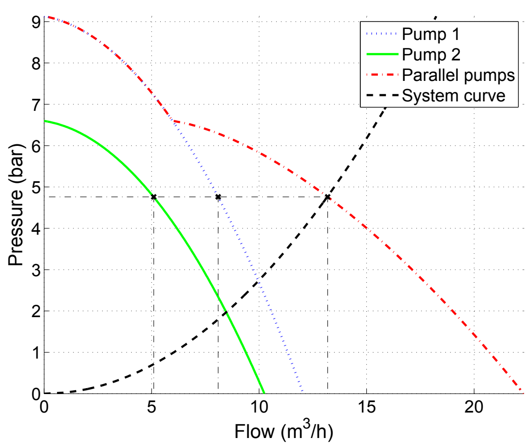

Figure 2: Operating points for pumps in a multi-pump system.

The Figure shows the system curve (dashed black). Where the system curve intersects the pump-curve for the multi-pump system (dash-dotted red), the system operating point is located. The operating points for the individual pumps (dotted blue and solid green) are then located at that head on their individual pump curves. This is shown in the figure as the intersection between the horizontal and vertical dash-dotted lines and the individual pump curves (dotted blue and solid green).

If it was desired for the pumps to remain at a given reference pressure, which is the normal situation for booster systems, then the addition of a pump should result in the speeds of the pumps to be decreased accordingly until the desired pressure was reached. The obtained flow for such a given pressure reference H_{ref} would then be

$$Q_{tot}(\vec{\omega}, H_{ref}) = \sum_{i=1}^{n} Q_i(\omega_i, H_{ref}) \ , \tag{12}$$

and the hydraulic power generated by the parallel pumps would be given by

$$P_{hyd_{tot}}(\vec{\omega}) = H_{ref} Q_{tot}(\vec{\omega}, H_{ref}) \ . \tag{13}$$

Note that H_{ref} is a pressure reference. If H_{ref} had been a head reference the formula in equation (13) should have been multiplied with $g \cdot \rho$.

The electrical power consumption is given by

$$P_{el_{tot}}(\vec{\omega}) = \sum_{i=1}^{n} P_{el_i}(\omega_i) \ . \tag{14}$$

The most efficient way of obtaining a given pressure reference H_{ref} for a multi-pump booster system would be to find the optimal combination of speeds for the available pumps. This optimal combination of speeds includes the situations where some of the pumps are switched off, since running one pump at a high speed could be more efficient than running two pumps at a lower speed. The overall efficiency of the system is given by

$$\eta_{tot} = \frac{P_{hyd_{tot}}(\vec{\omega})}{P_{el_{tot}}(\vec{\omega})} \ . \tag{15}$$

It should now be clear what the goal of this paper is. Based on a given reference pressure H_{ref} it is desired to find the combination of speeds for the parallel pumps that can generate this pressure while minimizing the energy cost. Before the minimization procedure can begin, it is first necessary to obtain the operating parameters for the pumps in the multi-pump booster system.

2.2.3 Estimation of Polynomial Coefficients

The polynomial coefficients are estimated using a simple genetic algorithm [3]. Since the three pumps used for the booster system are identical, the estimation is performed for only one of the pumps and then assumed to be valid for the remaining pumps as well. In short, the genetic algorithm uses a set of measurements from one of the pumps in order to estimate the four parameters a_0, b_0, c_0, and ω_0 from equation (5). The measurements are then used along with the affinity laws to minimize the estimation error of the pump speed ω using an implementation of a simple genetic algorithm toolbox for MATLAB made by Kumara Sastry [4]. The best set of pump curves are obtained using the parameters: $\mathbf{a_0 = -0.04}$, $\mathbf{b_0 = -0.00}$, $\mathbf{c_0 = 6.37}$, and $\boldsymbol{\omega_0} \mathbf{= 10.00}$, in the ranges $a_0 \in [-10, 0]$, $b_0 \in [-10, 0]$, $c_0 \in [0, 10]$, and $\omega_0 \in [6, 10]$.

The ranges were chosen based on various experiments in [3]. Especially the range for ω_0 should be noticed, as it was chosen based on the expectation that the pumps in the multi-pump system would operate primarily in this range. The approximation of the pump-curves using the affinity laws are only approximations, and in order to minimize the approximation error in the expected operating range the selected range was chosen. Further, it should be noticed that the speed ω is expressed using the reference voltage to the pump, which is in the range $[0, 10]$, because no measurements of the pump speeds were available.

The estimation of the pump-curves along with the measurements are shown in Figure 3.

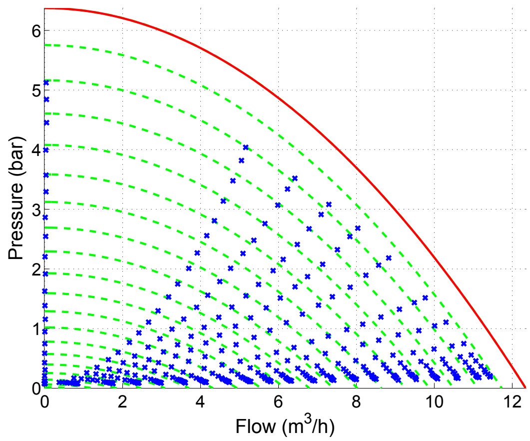

Figure 3: Best estimated pump-curves (base curve: solid red, estimated curves: dashed green, measurements: blue points).

Along with the measurements used for the estimation of the pump-curve parameters, a set of measurements of the power consumption was made. This is shown in Figure 4 as well as a plot of the best estimated power curve (solid red).

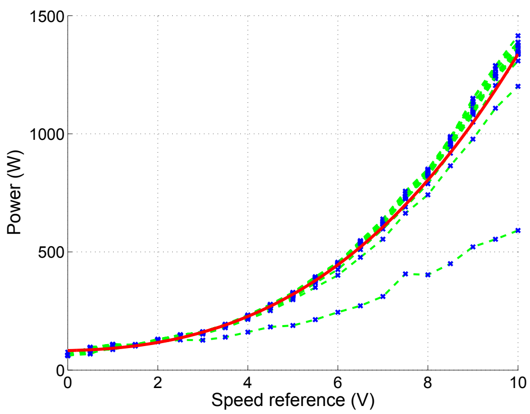

Figure 4: Measured power curves for different system loads (measurements: blue points, interpolations between measurements for constant system loads: dashed green) as well as best estimated power curve (solid red).

It can be seen from the figure that the power consumption is not fully independent of the system load. When the pressure in the pipes is high, the load on the pump motor is quite high. In that case it is assumed that the motor controller keeps the speed lower than the reference signal to the pump indicates resulting in a lower power consumption. In order to obtain precise results for situations of high pressure it is necessary to take this effect into account. However, in this paper it is decided to neglect this effect. It is thus assumed that the power consumption for a given reference speed is independent of the system load. It is then possible to estimate the power consumption purely as a function of the speed reference.

The values for the best estimated power curve are: $\mathbf{a_{el} = 0.7}$, $\mathbf{b_{el} = 5.4}$, $\mathbf{c_{el} = 3.8}$, and $\mathbf{d_{el} = 82.6}$, where the power curve is of the form given in equation (8).

The estimated power curve is a little imprecise at high speeds due to the inclusion of all measurement points when performing the estimate. All points have been included because the perceived system load of a single pump in the system can be quite high. It is then not unlikely that the actual speed, and thus also the power consumption, is lower than the reference would otherwise produce. So even though the effect of the motor controller is not accommodated for, it is still partly included in the estimate. The obtained results will thus become a little more imprecise when the reference pressure is set to low values.

Before the estimation of the optimal speed settings can begin it is necessary to take a look at the objective functions that can produce the desired results.

2.3 Objective Functions

It is desired to obtain the optimal speed settings of the multi-pump system for different loads and different reference pressures. In order to keep the estimation as simple as possible it is decided to fix the reference pressure at a given value before optimization and the optimization algorithm should then find the best possible combination of pump speeds for different system loads.

Instead of using the system load directly in the optimization process, it is possible to use the relationship given by the pump-curve. For a constant pressure, a change in system load will result in a change of flow. Thus, the flow Q_{tot} can be used in the optimization algorithm as an indicator for different system loads.

Since it is desired to minimize the power consumption it is straightforward to use the power consumption $P_{el_{tot}}$ as an optimization criterion. The objectives can thus be listed as

$$f_1 = \max Q_{tot}(\vec{\omega}, H_{ref}) \qquad (16)$$

$$f_2 = \min P_{el_{tot}}(\vec{\omega}) \ . \qquad (17)$$

It is quite easy to find f_2. Since the total power consumption is given by the sum listed in equation (14) it is just a matter of estimating the power consumption for the individual pumps given specific speed references.

An expression estimating the power consumption for a single pump given a specific reference speed has already been given in equation (8). The corresponding optimal parameters were given in Section 2.2.3. It is thus just a matter of implementing the mentioned equations in the optimization algorithm in order to obtain f_2. A special case occurs for a pump speed of 0 where the algorithm should consider the pump as turned off and the power should thus be set to 0.

Finding f_1 is a different matter. As indicated in equation (10), the objective function needs to estimate the flow contribution Q_i from each pump i. The pump-curve in equation (5) gives the relationship between the pressure and flow of a pump, and the corresponding parameters for the specific pumps used are given in Section 2.2.3. However, there are some constraints on the expression given by equation (5) before it can yield a corresponding flow to a given reference pressure.

The first constraint is when the reference pressure is higher than the amount of pressure the pump can deliver at a given speed. The constraint can be expressed in terms of the determinant of the second order polynomial for the pump-curve

(5), and can be written as

$$c_1 = \left(b_0 \frac{\omega_i}{\omega_0}\right)^2 - 4a_0 \left(c_0 \left(\frac{\omega_i}{\omega_0}\right)^2 - H_{ref}\right) \quad . \tag{18}$$

If $c_1 \leq 0$ then the referenced pressure is too high and the flow Q_i for pump i must be set to 0.

The second constraint is used to determine whether an intersection between the pump-curve and the reference pressure occurs for positive values of Q_i. Since the pump-curve is expressed as a second order polynomial with the top point located in the left half plane it is possible that a given reference pressure will intersect the pump-curve in the left half plane only. This constraint can be written as

$$c_2 = b_0 \frac{\omega_i}{\omega_0} + \sqrt{\left(b_0 \frac{\omega_i}{\omega_0}\right)^2 - 4a_0 \left(c_0 \left(\frac{\omega_i}{\omega_0}\right)^2 - H_{ref}\right)} \tag{19}$$

$$= b_0 \frac{\omega}{\omega_0} + \sqrt{c_1} \quad . \tag{20}$$

The first thing that should be noticed about c_2 is that it will only be non-complex if $c_1 \geq 0$. However, if that is not the case, then c_1 has already determined that the flow Q_i should be 0. Now, if $c_2 > 0$ then the intersection between the reference pressure and the pump-curve will occur for positive values of Q_i and the flow can be determined by

$$Q_i = \frac{-b_0 \frac{\omega_i}{\omega_0} - \sqrt{\left(b_0 \frac{\omega_i}{\omega_0}\right)^2 - 4a_0 \left(c_0 \left(\frac{\omega_i}{\omega_0}\right)^2 - H_{ref}\right)}}{2a_0} \tag{21}$$

$$= \frac{-c_2}{2a_0} \quad . \tag{22}$$

If $c_2 \leq 0$ then the intersection occurs only for negative values of Q_i and it should be set to 0.

In order to find f_1 it is thus necessary to implement equations (18), (20), (22), and (10).

For the cases when a pump has insufficient speed to contribute to the flow of the system the power consumption for that pump would be excessive. The optimization algorithm should then be able to accommodate for this and either turn the speed of the pump up, such that it can contribute to the flow, or by turning the pump off, which is done by setting the pump speed to 0.

In order to avoid problems when calculating the efficiency, a minimum value for the power consumption of $2.2 \cdot 10^{-16}$ will be used if all three pumps are set to speed 0

With these fitness functions in place it is just a matter of setting the different run-time parameters for the MOEA and running it to obtain the results.

3. NSGA-II

The non-dominated sorting genetic algorithm (NSGA-II) [1] was chosen for this multi-objective optimization problem as it is a widely used and capable algorithm. However, in order to make use of MATLAB for performing the objective function calculations and for plotting the results, an implementation of that algorithm as a toolbox for MATLAB made by Kumara Sastry [4] was used. A list of the run-time parameters used for the optimization algorithm is given in Table 1.

The population size and the maximum number of generations are chosen significantly higher than the default values of 106 and 26 respectively. The reason for this is that some preliminary runs with the default values resulted in a significant amount of noise on the speed settings of the pumps. It was suspected that the non-dominated solutions were not Pareto optimal and that an increase of both population size and maximum number of generations would allow more solutions on or very close to the Pareto front to be found.

Table 1: Parameters used for running NSGA-II.

Parameter	Setting
Population size	200
Maximum no. of generations	100
Representation type	Real values
Selection strategy	Tournament
Tournament size	2
Crossover type	SBX
Crossover probability	0.9
SBX parameter	10
Mutation type	Polynomial
Mutation probability	0.1
Polynomial parameter	20

Real valued representation was chosen arbitrarily, as it was not expected that a binary representation would produce better results. Further, it meant that it was not necessary to worry about the size of the genome.

For selection, crossover, and mutation the default parameters of the MATLAB toolbox were used.

Before using the algorithm to find the optimal speed settings for the individual pumps in the multi-pump system, there were a few more issues that needed to be decided upon.

3.1 Implementation

The first thing that needed to be decided was how many different reference pressures the algorithm should be tested with, and also what particular reference pressures should be chosen. After some investigations it was decided to run the optimization algorithm with three different reference pressures, $H_{ref} = 2, 3, 4$.

Further, it was considered whether the algorithm should be tested with different pump-curve approximations than the one given in Section 2.2.3. In order to keep the presented results at an acceptable number it was decided to only use the pump-curve approximation presented previously. It is expected that a variety of results could be obtained for other pump-curve approximations and even other pumps.

Finally, it was decided to further reduce the noise in the obtained results by performing a non-dominated check of all the individuals encountered during the optimization process. As such, the number of non-dominated solutions that will be presented can be quite numerous.

So, with the various issues decided upon it is time to look at the results obtained for the conditions described.

4. RESULTS

The optimization algorithm was first run for a reference pressure of 2 bar.

4.1 Reference Pressure of 2 bar

The results of the optimization can be seen in Figure 5, where the horizontal axis represents the flow f_1 and the vertical axis represents the electrical power consumption f_2

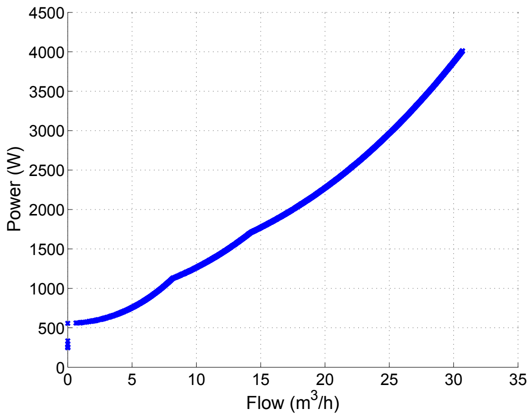

Figure 5: Power consumption for the multi-pump system for a reference pressure, H_{ref}, of 2 bar.

The Utopian point in the figure is located in the lower right hand corner, where flow is maximized and power consumption minimized. It can be seen that the non-dominated solutions generate a nice continuous curve with a couple of sharp bends near $Q_{tot} = 8$ and $Q_{tot} = 14$. In order to better see the effect of the optimization it is worth taking a look at a plot of the efficiency shown in Figure 6.

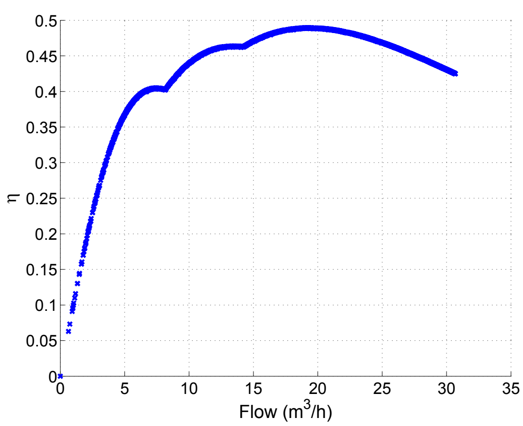

Figure 6: Efficiency of the multi-pump system for a reference pressure, H_{ref}, of 2 bar.

Once the total flow Q_{tot}, the total electrical power consumption $P_{tot_{el}}$, and the reference pressure H_{ref} are known, the efficiency can easily be calculated using equation (15). The plot of efficiency also shows a nice continuous curve with two sharp bends near the values $Q_{tot} = 8$ and $Q_{tot} = 14$. It seems like the obtained curve is made up as the maximum of three separate curves that intersect. This is further illustrated since it can be seen that the curve has three local maxima.

The figure also shows that at no point does the efficiency of the multi-pump system reach 0.5 which indicates that over 50% of the energy contributed to the system will be lost and not used to pump the liquid.

The most interesting thing about the optimization performed is how the distribution of speeds for the individual pumps turns out. This is shown in Figure 7.

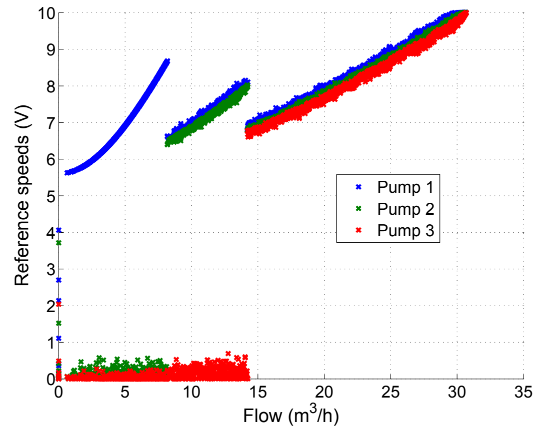

Figure 7: Speed settings for the individual pumps at a reference pressure, H_{ref}, of 2 bar.

One thing should be emphasized when looking at the figure. Since the three pumps are identical, the allocation of speeds are arbitrary and at any given point of the graph the pump allocated could just as easily have been one of the others. In order to get a better overview of the results it was decided to sort the results in such a way that pump 1 would always contribute with the highest speed, pump 2 with the second highest speed, and pump 3 with the lowest speed. Had the pumps not been identical this sorting would not be appropriate, but then the resulting speeds would not have been arbitrarily distributed amongst the pumps.

The figure shows a clear tendency. When the flow is lower than $Q_{tot} = 8$, only one pump is used and the remaining two pumps are off. The figure does show some minor speeds for the other two pumps, but these are considered to be noise as the speed distribution of those pumps are quite noisy. Also, those speeds were reduced significantly from the preliminary runs where the non-dominated solutions were farther from the Pareto optimal front. So, if enough computation was used to discover the entire Pareto optimal set it is expected that these speeds would continue towards 0.

Around $Q_{tot} = 8$ it is seen that the second pump is started and the speed of the first pump is reduced such that there now are two pumps running at approximately the same speed. This agrees well with the previous results plotted in Figures 5 and 6. This tendency continues until around $Q_{tot} = 14$ where the third pump is turned on and the speeds of the other pumps are reduced such that they all run at approximately the same speed. This continues until the maximally obtainable flow can be achieved.

It is thus evident that the most efficient distribution of speeds is to have the pumps running at the same speed once they are turned on and that distinct flow rates indicate when it is time to either turn the pumps on or off.

With the results of the 2 bar reference pressure presented the results for a reference pressure of 3 bar will now be given.

4.2 Reference Pressure of 3 bar

The results for running the optimization algorithm with a reference pressure of $H_{ref} = 3$ differs somewhat from the results for the lower reference pressure of 2 bar.

The total flow and electrical power consumption for the optimal allocation of pump speeds is shown in Figure 8.

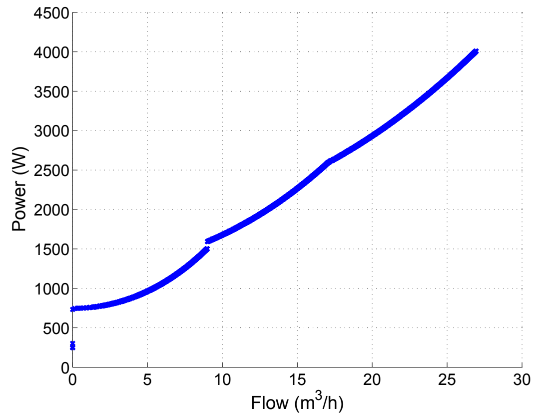

Figure 8: Power consumption for the multi-pump system for a reference pressure, H_{ref}, of 3 bar.

This time the non-dominated solutions do not form a continuous curve. At around $Q_{tot} = 9$ it is seen that the power consumption jumps a little thus creating a discontinuity. Further it is seen that the curve also has a sharp bend around $Q_{tot} = 17$ which would indicate that a change in the number of active pumps takes place here.

Before looking at the speeds, it is interesting to take a look at the efficiency shown in Figure 9.

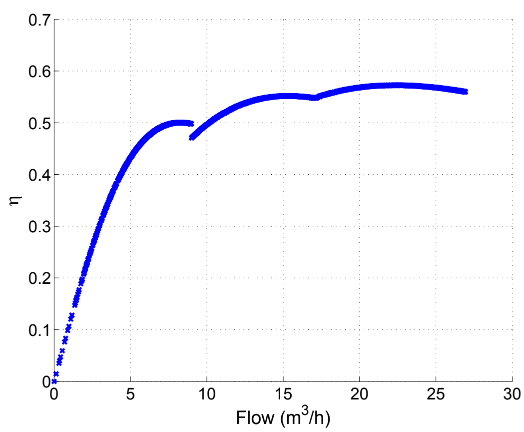

Figure 9: Efficiency of the multi-pump system for a reference pressure, H_{ref}, of 3 bar.

The efficiency curve confirms the observations from the plot of the power consumption. At around $Q_{tot} = 9$ there is a significant drop in efficiency. However, at around $Q_{tot} = 17$ it seems like an intersection between two different curves takes place, which is similar to what was seen for the lower reference pressure of 2 bar. Contrary for the previous case, it is seen that the efficiency for this reference pressure is significantly better. For high flow rates the efficiency exceeds $\eta = 0.55$.

The speed settings for this reference pressure is given in Figure 10.

Once again it can be seen that the speed changes happen where the efficiency and power consumption plots have sharp bends or discontinuities. Also, when the pumps are running they are running at approximately the same speeds,

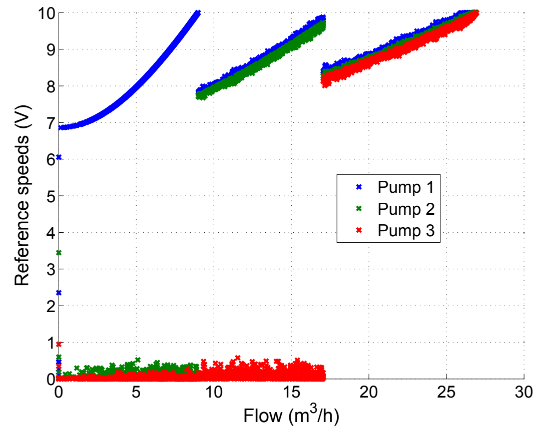

Figure 10: Speed settings for the individual pumps at a reference pressure, H_{ref}, of 3 bar.

indicating that the most efficient speed setting is to run the active pumps equally. The reason for the discontinuity of the previous plots can also be seen. At around $Q_{tot} = 9$, the running pump reaches the maximum speed, and in order for the multi-pump system to generate a flow just above that setting, it is necessary to turn on an additional pump. This results in the jump of power consumption since two pumps require more power than a single pump would have required, if such a single pump could have delivered enough flow.

It is now time to take a look at the final setting for the reference pressure.

4.3 Reference Pressure of 4 bar

The last reference pressure setting that is investigated is for $H_{ref} = 4$. The obtained non-dominated front that shows the power consumption can be seen in Figure 11.

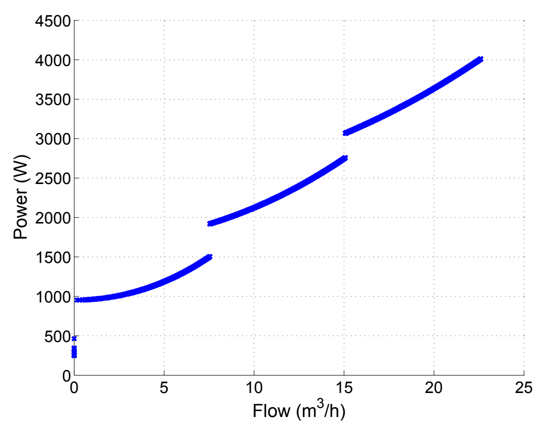

Figure 11: Power consumption for the multi-pump system for a reference pressure, H_{ref}, of 4 bar.

This time it can be seen that the power consumption curve clearly consists of three separate segments since there are two distinct discontinuities at $Q_{tot} = 7.5$ and $Q_{tot} = 15$. It is expected that this is because the speeds of the pumps reach the maximal values before it would be more efficient to add an additional pump.

The plot of the efficiency can be seen in Figure 12.

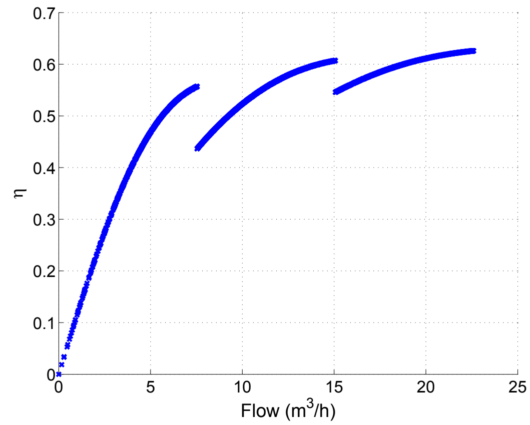

Figure 12: Efficiency of the multi-pump system for a reference pressure, H_{ref}, of 4 bar.

The efficiency plot also shows the discontinuities that the power consumption plot showed. However, it also shows that the efficiency curves are still rising when the discontinuity occurs. Even for the maximal flow rate it can be seen that the curve is rising. This would indicate that the pumps are too weak to satisfactorily deliver the most efficient performance for a system requiring this pressure reference. This can be concluded since the best efficiency point on any of the curve segments is never reached. However, the efficiency does reach past $\eta = 0.6$ in several places.

Finally, a plot of the speed settings is shown in Figure 13.

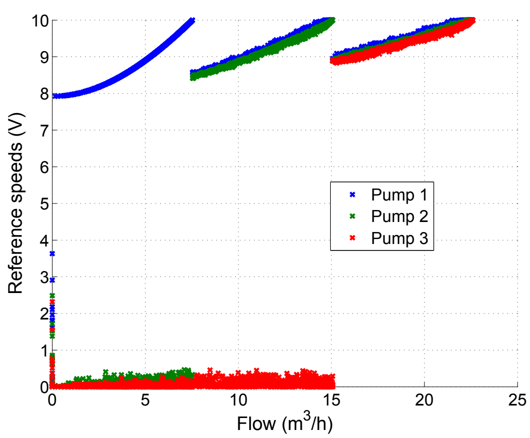

Figure 13: Speed settings for the individual pumps at a reference pressure, H_{ref}, of 4 bar.

The figure again shows that when more than one pump is active the pumps should run at the same speed. It is also confirmed that the speeds of the pumps reach the maximum value before a new pump is turned on. It can further be seen that the pumps are constantly given reference speeds around 8 or more leaving very little room for adjustments. So it is quite clear that the system will be somewhat stressed with a reference pressure at 4 bar.

With the results presented it is time to make a few conclusions.

5. CONCLUSION

This paper presented a way to find the near-optimal speed settings for a multi-pump booster system given a specific reference pressure. Two objectives were presented, one to be minimized and one to be maximized, which produced a set of non-dominated solutions.

The optimization of the speed settings was performed for three different reference pressures which produced some interesting results.

The most characteristic result obtained is that the most efficient way of running the multi-pump booster system is to have the active pumps running at the same speed. If the multi-pump booster system was modified to include several different pump types this might of course not be the case, but the optimization algorithm should still be able to find a set of near-optimal speed settings.

It was also seen that as the reference pressure was increased the system had a harder time with reaching the most efficient operating points. However, at higher pressure conditions the overall efficiency did reach higher values than for low pressure reference conditions. Also, based on the efficiency plot it could be determined whether the multi-pump booster system was adequately suited to run with a given pressure reference.

Several improvements can be made to the current optimization algorithm. First of all it could include measures that will take the load conditions of the pumps under consideration. This would improve the accuracy of the algorithm for high pressure conditions. Further, it would be advantageous to test the results obtained on the physical multi-pump booster system to ensure that the obtained results are accurate and valid.

It is the belief of the authors that this optimization algorithm has great potential for optimizing the speed settings of any multi-pump system such that it can reach the most efficient working conditions.

6. ACKNOWLEDGMENTS

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