

# Efficiency Optimization of a Multi-Pump Booster System

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## ABSTRACT

This paper discusses a way to optimize the speed settings for a multi-pump system such that it can operate with highest efficiency. A short description of an actual multi-pump system is given and the mathematical formulas for single pump and multi-pump systems are presented. Then a set of objectives are formulated which, when used in a MATLAB toolbox implementation of NSGA-II, describe the most efficient distribution of speeds amongst the pumps in the system. The system is tested for a number of pressure references with good results.

## Categories and Subject Descriptors

J.2 [Computer Applications]: Physical Sciences and Engineering; J.6 [Computer Applications]: Computer-aided engineering—*Computer-aided design*

## General Terms

Performance, Design

## Keywords

Pump optimization, Parallel pumps, Multi-objective optimization, NSGA-II

## 1. INTRODUCTION

Pumps are some of the most used electrical components worldwide [2]. Whenever a liquid needs to be moved from one place to another, and it is not possible to just let gravity handle the transfer, a pump is used. Since the use of pumps is so widespread it is important to keep the efficiency in mind when choosing a pump for a specific application. If not, then a lot of energy will quickly go to waste.

The first step in optimizing efficiency of pumps can be made by varying the speeds at which they run. A pump that runs at a constant speed without regard to the load of the system connected to it has to be chosen such that it

can handle the worst case pressure/flow scenario. That also means that for non-worst case running scenarios it will be running too fast and will be wasting energy since the excess energy only will raise the pressure of the system.

Just varying the speed of a pump is not sufficient to optimize the operation of a pump since the efficiency of these variable speed pumps varies with speed. Depending on the type of pump, a variable speed pump is most efficient at a specific speed and certain pressure/flow conditions. In fact, the type name of pumps usually include specifications of the optimal flow conditions.

When pumping liquids it is commonplace to control the increase in pressure of the liquid. Depending on the load of the system, the speed of the pump is varied to achieve the desired pressure. However, for some applications there can occur large variations in the flow of the liquid, including water supply for high-rise buildings and water supply for fire extinguishing. In such cases a pump will often operate in ranges where the efficiency of the pump is significantly lower than the optimally rated conditions. It might be more efficient to use several smaller pumps instead of using a single large pump. The problem then is to determine the best combination of speeds for the small pumps such that the overall efficiency of the pumping unit is as high as possible.

This paper investigates how the most efficient combination of speeds for a multi-pump booster system can be found using a Multi-Objective Evolutionary Algorithm (MOEA). Before reaching that goal it is necessary to first take a look at such a multi-pump booster system. In Section 2 a description of the system is given, both physically as well as a mathematical model. The mathematical model in Section 2.2 also explains how the equations for a single pump can be combined to describe a system with several pumps in parallel. In that section, the coefficients relating the mathematical model to the physical system are also given.

In Section 2.3, the objective functions describing the efficiency of the system is given and in Section 3 the MOEA used to solve the problem (NSGA-II)[1] is described along with implementation details. The obtained results are shown in Section 4 followed by a conclusion in Section 5.

## 2. MULTI-PUMP BOOSTER SYSTEM

This section will give an introduction to the physical multi-pump booster system which is used as case study in this paper. Following that a mathematical description of a single and multiple pumps in parallel are presented along with the corresponding coefficients for the system under consideration.

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## 2.1 Physical System

The physical system consists of 3 CRE5-8 pumps mounted in parallel between an inlet pipe and an outlet pipe. The speed of each pump can be controlled independently of the others and the power consumed by each pump can be monitored. Further, measurements of the total flow in the system,  $Q_{tot}$ , as well as the pressure difference from intake pipe to outlet pipe,  $\Delta p$ , are available. It is thus possible to perform measurements on each individual pump as well as the entire system such that the coefficients of the mathematical model, which is described next, can be calculated.

## 2.2 Mathematical Model

The mathematical model of the system is going to be used as the basic element in the optimization process since it will be performed offline and should result in a set of optimal speeds for the multi-pump system under different circumstances. First, a mathematical model of one of the pumps will be given and then extended to the case for multiple pumps in parallel.

Before starting with the mathematical model of the pump it is important that the technical terms used in connection with pumps are clear to the reader. The main difference is how to express the pressure generated by the pumps. Instead of using pressure difference  $\Delta p$  to indicate the increase in pressure from inlet to outlet it is common to use the term head given by

$$H = \frac{\Delta p}{\rho g} , \quad (1)$$

where  $g$  is the gravity constant and  $\rho$  is the density of the liquid being pumped. The reason for this change is that the head represents the height to which a non-specified liquid can be pumped. The equations thus become independent on the liquid in the system and the thermal properties of said liquid as well.

### 2.2.1 Single Pump

The mathematical model for a pump is a static model showing the relationship between the head and flow for different system loads. Due to various losses the relationship between head and flow are normally given by a second-, third-, or fourth-order polynomial with regard to the flow,  $Q$ . In this particular case a second order polynomial will be used resulting in the following pump curve that varies according to different pump speeds,  $\omega$ .

$$H(\omega) = a(\omega)Q(\omega)^2 + b(\omega)Q(\omega) + c(\omega) . \quad (2)$$

The coefficients  $a(\omega)$ ,  $b(\omega)$ , and  $c(\omega)$  vary according to the speed  $\omega$  but can be approximated using the affinity laws [2]

$$Q(\omega_B) = \frac{\omega_B}{\omega_A} Q(\omega_A) \quad (3)$$

$$H(\omega_B) = \left(\frac{\omega_B}{\omega_A}\right)^2 H(\omega_A) . \quad (4)$$

These are good approximations to the actual change in pump curve when the speed changes, provided the system curve remains unchanged. The approximated pump curve then becomes

$$H(\omega) = a_0 Q(\omega)^2 + b_0 \frac{\omega}{\omega_0} Q(\omega) + c_0 \left(\frac{\omega}{\omega_0}\right)^2 , \quad (5)$$

where  $a_0$ ,  $b_0$ , and  $c_0$  are the coefficients for the pump curve at the base speed  $\omega_0$ . Thus, if  $a_0$ ,  $b_0$ ,  $c_0$ , and  $\omega_0$  are known then an approximate pump curve can be calculated for any speed  $\omega$ .

The amount of power delivered from the pump to the liquid is called the hydraulic power and is given by

$$P_{hyd}(\omega) = H(\omega)g\rho Q(\omega) . \quad (6)$$

For hydraulic power the affinity law yields

$$P_{hyd}(\omega_B) = \left(\frac{\omega_B}{\omega_A}\right)^3 P_{hyd}(\omega_A) , \quad (7)$$

which indicates that the power consumption of the pump depends on the speed to the third power. So, when measuring the electrical power consumption of the pump, which includes losses in the pump and motor as well as the hydraulic power, it should be expected that the power depends on the speed to the third power. The mathematical model of the pump should thus include a third order polynomial representing the power consumption of the pump which can be based on measurements of the actual pump under consideration. The expression for the power consumption can thus be expressed as

$$P_{el}(\omega) = a_{el}\omega^3 + b_{el}\omega^2 + c_{el}\omega + d_{el} . \quad (8)$$

The efficiency of a pump is given by the relationship between the amount of power delivered to the liquid and the total power consumed,

$$\eta = \frac{P_{hyd}(\omega)}{P_{el}(\omega)} . \quad (9)$$

With the mathematical model of a single pump defined it is now time to look at the mathematical model for multiple pumps in parallel.

### 2.2.2 Multiple Parallel Pumps

When several pump are connected in parallel the total head is identical to the head generated by each individual pump, whereas the total flow in the system is the sum of the flows for each pump. So, if a system consists of  $n$  pumps the total flow of the system will be

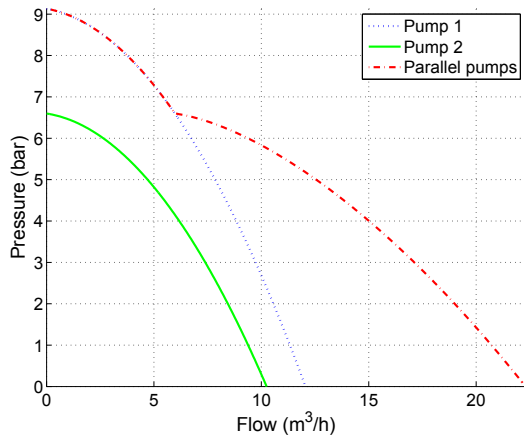
$$Q_{tot}(\vec{\omega}) = \sum_{i=1}^n Q_i(\omega_i) , \quad (10)$$

where  $\vec{\omega}$  is a vector containing the speeds  $\omega_i$  of each pump  $i$ , and the head will be

$$H_{tot}(\vec{\omega}) = H_1(\omega_1) = \dots = H_n(\omega_n) . \quad (11)$$

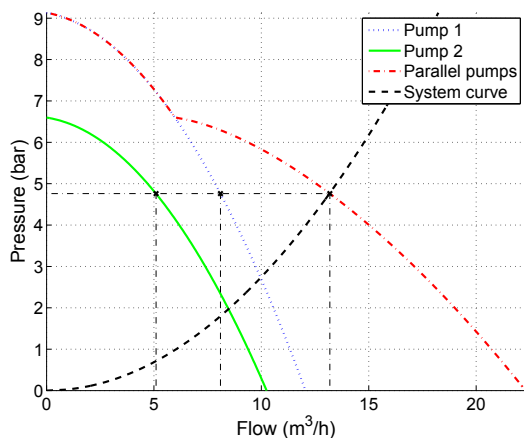
A graphical representation of the pump curve resulting from putting several pumps in parallel is given in Figure 1. Note that the pump-curves are based on actual pumps in the multi-pump system and have not been illustrated with head but with pressure when pumping water.

The effect of adding additional pumps to a system in parallel does not mean that only the flow will increase. If the system load is unchanged an addition of a pump will result in an increase of both flow and pressure. This is because the flow of liquid contributed from the extra pump will take up some of the space in the pipes of the system. The original pump(s) will thus see the addition of the extra pump as a



**Figure 1: Pump curve for individual pumps and multiple pumps in parallel.**

narrowing of the pipes, which corresponds to a higher system load. The contribution from the original pump(s) will then have a higher pressure and less flow. The extra pump will, however, also be operating at this increased head and still yield additional flow. This is illustrated in Figure 2.



**Figure 2: Operating points for pumps in a multi-pump system.**

The Figure shows the system curve (dashed black). Where the system curve intersects the pump-curve for the multi-pump system (dash-dotted red), the system operating point is located. The operating points for the individual pumps (dotted blue and solid green) are then located at that head on their individual pump curves. This is shown in the figure as the intersection between the horizontal and vertical dash-dotted lines and the individual pump curves (dotted blue and solid green).

If it was desired for the pumps to remain at a given reference pressure, which is the normal situation for booster systems, then the addition of a pump should result in the speeds of the pumps to be decreased accordingly until the desired pressure was reached. The obtained flow for such a

given pressure reference  $H_{ref}$  would then be

$$Q_{tot}(\vec{\omega}, H_{ref}) = \sum_{i=1}^n Q_i(\omega_i, H_{ref}) , \quad (12)$$

and the hydraulic power generated by the parallel pumps would be given by

$$P_{hyd_{tot}}(\vec{\omega}) = H_{ref} Q_{tot}(\vec{\omega}, H_{ref}) . \quad (13)$$

Note that  $H_{ref}$  is a pressure reference. If  $H_{ref}$  had been a head reference the formula in equation (13) should have been multiplied with  $g \cdot \rho$ .

The electrical power consumption is given by

$$P_{el_{tot}}(\vec{\omega}) = \sum_{i=1}^n P_{el_i}(\omega_i) . \quad (14)$$

The most efficient way of obtaining a given pressure reference  $H_{ref}$  for a multi-pump booster system would be to find the optimal combination of speeds for the available pumps. This optimal combination of speeds includes the situations where some of the pumps are switched off, since running one pump at a high speed could be more efficient than running two pumps at a lower speed. The overall efficiency of the system is given by

$$\eta_{tot} = \frac{P_{hyd_{tot}}(\vec{\omega})}{P_{el_{tot}}(\vec{\omega})} . \quad (15)$$

It should now be clear what the goal of this paper is. Based on a given reference pressure  $H_{ref}$  it is desired to find the combination of speeds for the parallel pumps that can generate this pressure while minimizing the energy cost. Before the minimization procedure can begin, it is first necessary to obtain the operating parameters for the pumps in the multi-pump booster system.

### 2.2.3 Estimation of Polynomial Coefficients

The polynomial coefficients are estimated using a simple genetic algorithm [3]. Since the three pumps used for the booster system are identical, the estimation is performed for only one of the pumps and then assumed to be valid for the remaining pumps as well. In short, the genetic algorithm uses a set of measurements from one of the pumps in order to estimate the four parameters  $a_0$ ,  $b_0$ ,  $c_0$ , and  $\omega_0$  from equation (5). The measurements are then used along with the affinity laws to minimize the estimation error of the pump speed  $\omega$  using an implementation of a simple genetic algorithm toolbox for MATLAB made by Kumara Sastry [4]. The best set of pump curves are obtained using the parameters:  $\mathbf{a}_0 = -0.04$ ,  $\mathbf{b}_0 = -0.00$ ,  $\mathbf{c}_0 = 6.37$ , and  $\omega_0 = 10.00$ , in the ranges  $a_0 \in [-10, 0]$ ,  $b_0 \in [-10, 0]$ ,  $c_0 \in [0, 10]$ , and  $\omega_0 \in [6, 10]$ .

The ranges were chosen based on various experiments in [3]. Especially the range for  $\omega_0$  should be noticed, as it was chosen based on the expectation that the pumps in the multi-pump system would operate primarily in this range. The approximation of the pump-curves using the affinity laws are only approximations, and in order to minimize the approximation error in the expected operating range the selected range was chosen. Further, it should be noticed that the speed  $\omega$  is expressed using the reference voltage to the pump, which is in the range  $[0, 10]$ , because no measurements of the pump speeds were available.









