

The mathematical model of the problem is:

$$\min z = \sum_{t=0}^T \left[\sum_{j=1}^J \sum_{m=1}^M c_{ij} x_{ij}(t) + \sum_{j=1}^J \sum_{k=1}^K \sum_{m=1}^M c_{jkm} x_{jkm}(t) + \sum_{k=1}^K \sum_{m=1}^M c_{kfm} x_{kfm}(t) - \sum_{k=1}^K \sum_{m=1}^M c_{krm} x_{krm}(t) + \sum_{k=1}^K \sum_{m=1}^M c_{kDm} x_{kDm}(t) + \sum_{m=1}^M c_{SFm} x_{SFm}(t) + \sum_{j=1}^J c_j^1 y_j^1(t) + \sum_{k=1}^K \sum_{m=1}^M c_{km}^2 y_{km}^2(t) + \sum_{j=1}^J c_j^{1op} z_j^1 + \sum_{k=1}^K c_k^{2op} z_k^2 \right] \quad (1)$$

$$\text{s. t. } x_{SFm}(t) + \sum_{k=1}^K x_{kFm}(t) \geq d_m, \quad \forall m, t \quad (2)$$

$$\sum_{j=1}^J x_{ij}(t) + y_j^1(t-1) \leq u_{ij} z_j^1, \quad \forall j, t \quad (3)$$

$$\sum_{k=1}^K x_{jkm}(t) + y_{km}^2(t-1) \leq u_{km} z_k^2, \quad \forall k, m, t \quad (4)$$

$$x_{krm}(t) \leq u_R, \quad \forall k, m, t \quad (5)$$

$$\sum_{j=1}^J z_j^1(t) \leq J, \quad \forall t \quad (6)$$

$$\sum_{k=1}^K z_k^2(t) \leq K, \quad \forall m, t \quad (7)$$

$$x_{ij}(t), x_{jkm}(t), x_{kfm}(t), x_{krm}(t), x_{kDm}(t), x_{SFm}(t) \geq 0 \quad (8)$$

$$z_j^1, z_k^2 \in \{0, 1\} \quad (9)$$

3. HYBRID GENETIC ALGORITHM

We here adopt the priority-based encoding method developed Gen, Altıparmak and Lin [3]. Although this encoding had been successfully applied on shortest path problem and project scheduling problem, the difference of our approach comes from the facts that of special decoding and encoding procedures for transportation trees. The priority-based encoding method is an indirect approach. In this method, a gene in chromosome contains two kinds of information: the locus, the position of the gene within the structure of a chromosome, and the allele, the value the gene takes. The position of a gene is used to represent a node (source depot), and the value is used to represent the priority of the node for constructing a tree among candidates.

For a transportation problem, a chromosome consists of priorities of sources and depots to obtain transportation tree and its length is equal to total number of sources m and depots n , i.e. $m+n$. The transportation tree corresponding with a given chromosome is generated by sequential arc appending between sources and depots. At each step, only one arc is added to tree selecting a source (depot) with the highest priority and connecting it to a depot (source) considering minimum cost.

For mLNTP, we use two priority-based encodings to represent the transportation trees on stages. This means that each chromosome in the population consists of two parts. While the first part (i.e. the first priority-based encoding) represents transportation tree between return centers and disassembly centers, the second part (i.e. the second priority-based encoding) represents transportation tree between disassembly centers and processing centers. The detailed encoding and decoding process of a chromosome consists of three main phase:

Phase 1: Coding process on 1st stage

Step 1.1: Priority-based encoding from returning center to disassembly center

Step 1.2: Decoding from returning center to disassembly center

Phase 2: Coding process on 2nd stage

Step 2.1: Priority-based encoding from disassembly center to processing center

Step 2.2: Decoding from disassembly center to processing center

Phase 3: A heuristic approach in the 3rd stage

1) **Crossover Operator:** We proposed a new crossover operator, weight mapping crossover (WMX). WMX is same as that of the conventional one-cut point crossover can generate two new paths that exchanged sub-route from two parents.

2) **Mutation Operator:** In this study, insertion mutation has been adopted. Insertion mutation selects a gene at random and inserts it in a random position.

4. EXPERIMENT AND DISCUSSION

We used using pnGA (Prüfer number-based GA) proposed by Syarif and Gen [4], to study the effectiveness of the developed GA with new encoding method (priGA). The number of returning centers changes between 4 and 30, number of disassembly centers and number of processing centers changed between 3-15, and 3-20, respectively. The transportation costs, demand of parts, capacities of returning centers, disassembly processes, processing processes, recycles and manufacturer are randomly generated to provide realistic scenarios.

Numerical experiments demonstrated the efficiency and effectiveness of the hybrid GA approach for solving the mLNTP problems. Although memory requirement for new representation was greater than pnGA, i.e., Prüfer number-based GA, only 2 digits for each stage in transportation problem, this representation got very important two advantages in the real world applications. One of them was that its implementation was very easy. Another one was that after genetic operators, always feasible solutions were obtained. Based on this study, it was seen that the hybrid priGA with WMX demonstrated the best performance according to solution quality.

Table. 1 Computational results with pnGA and priGA

Problems No.	stage	no. of constraints	no. of shipping variables	pnGA			priGA		
				Best	Average	ACT	Best	Average	ACT
1	stage 1	1275	975	35650	35650	0.65	35650	35650	0.65
	stage 2			43200	43200	0.60	43200	43200	0.65
	stage 3			50100	50100	-	50100	50100	-
	total			128950	128950	1.25	128950	128950	1.30
2	stage 1	2505	2055	53795	54710	0.90	53490	54250	1.00
	stage 2			48270	50180	0.95	47370	49800	0.95
	stage 3			75500	75500	-	75500	75500	-
	total			177565	180390	1.85	176360	179550	1.95
3	stage 1	8745	7800	86300	88006	6.10	82920	85273	6.05
	stage 2			81480	81661	6.80	80730	83223	6.00
	stage 3			110050	110050	-	110050	110050	-
	total			277830	279717	12.90	273700	278546	12.05
4	stage 1	20430	18930	195895	199381	13.05	189765	193995	12.30
	stage 2			198915	205788	9.40	192900	199822.5	10.75
	stage 3			237675	237675	-	237675	237675	-
	total			632485	642844	22.45	620340	631492.5	23.05

5. REFERENCES

- [1] Stock, J. K., 1992. Reverse logistics, White Paper, Council of Logistics Management, Oak Brook, IL.
- [2] Gen, M. and Cheng, R. W., 1997. Genetic Algorithm and Engineering Design, Wiley, New York.
- [3] Gen, M., Altıparmak, F. and Lin, L., 2006. A genetic algorithm for two-stage transportation problem using priority-based encoding, OR Spectrum, 28(3), 337-354.
- [4] Syarif, A. and Gen, M. 2003. Double Spanning Tree-based Genetic algorithm For Two Stage Transportation Problem, International Journal of Knowledge-Based Intelligent Engineering System, 7(4), 388-389.