Interplanetary Trajectory Optimization with Swing-bys Using Evolutionary Multi-Objective Optimization

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ABSTRACT

Interplanetary trajectory optimization studies mostly considered a single objective of minimizing the travel time between two planets or the launch velocity of spacecraft at the departure planet. In this paper, we have considered a simultaneous minimization study of both launch velocity and time of travel between two specified planets with and without the use of gravitational advantage (swing-by) of some intermediate planets. Using careful consideration of a Newton-Raphson based root finding procedure of developing a trajectory based on a given set of decision variables (departure date, swing-by planets, altitude of spacecraft at the first swing-by planet, etc.), a number of derived parameters such as time of flight between arrival and destination planet, date of arrival, and launch velocity are computed. A popularly used evolutionary multi-objective optimization algorithm (NSGA-II) is then employed to find a set of trade-off solutions. The accuracy of the developed software (we called GOSpel) is first demonstrated by matching the trajectories with known missions and then the efficiency of the software is shown by solving a number complex, real-world like missions.

Categories and Subject Descriptors: G.1.6 [Numerical Analysis]: Optimization

General Terms: Algorithms

1. INTRODUCTION

The interplanetary mission design is a challenging task. As spacecraft travels through our solar system it may encounter many celestial bodies, and may get influenced by their gravitational fields (swing-by of planets). Due to these effects, spacecraft may deviate from its path or even may get damaged. On the other hand, these gravitational fields may be used in a constructive way to reduce energy requirement of a flight. Sending satellites to interplanetary trajectory is risky and expensive. There can be various trajectories which a spacecraft may follow. But there has to be an optimal trajectory which when followed, gives high performance boost either in energy requirement or in time required for the mission.

Genetic Algorithms (GAs) have been used for over 20 years in various applications of optimization. GAs are successfully applied in many complex real-world optimization problems where the function to be optimized is highly non-linear and discrete. In the recent past, they have been adapted very successfully in solving multi-objective optimization problems involving more than one conflicting objectives. Non-dominated Sorting Genetic Algorithm-II (NSGA-II) [4] and Strength Pareto Evolutionary Algorithm (SPEA-II) [8] are examples of such GA based multi-objective optimizers. The present work explores the possibility of multi-objective optimization in interplanetary trajectory optimization. It highlights the efficiency of GAs to find almost all possible solutions with varying properties which leads to observational principles about the trends and trade-offs. The most interesting role of the developed software, GOSpel, lies in not, just solving the problem but also optimizing it in a conflicting multi-objective scenario. Input parameters like, the time of window in which optimal solutions are sought, type of the transfer, choice of mission and information about the swing-by planets are accepted from the user. The practical limits on launch parameters like difference in flyby velocities, maximum and minimum bounds on number of days can also be specified by the user.

GECCO'08, July 12–16, 2008, Atlanta, Georgia, USA.
ACM 978-1-60558-131-6/08/07.

2. INTERPLANETARY TRAJECTORY OPTIMIZATION

In general, the interplanetary trajectory design involves three major phases: (i) departure hyperbolic trajectory phase relative to departure planet, (ii) interplanetary transfer trajectory phase relative to central body (that is, Sun) and (iii) approach trajectory phase relative to the arrival or intermediate swing-by planet. These three phases must be synchronized to realize a mission. A large number of iterations are required to be carried out on these three phases to synchronize them. Trajectory planning is dependent on the kind of mission which is to be pursued, namely, which type of mission is it? Is it an orbiter mission or a flyby mission? An orbiter mission requires the satellite to reach the destination planet with a particular velocity so that it continues to revolve about the planet under it's gravitational effect. Swing-by means moving towards an intermediate planet on a hyperbolic trajectory, coming close to it and without colliding recede back again on a hyperbolic path. In an effective swing-by, the planet's gravitational pull may be assisted to maneuver satellite towards the destination planet. If gravity assistance (swing-by) from more than one planets used then it is important to know how many times and from which planet such an assistance is used? In finding the optimal trajectory, all the above factors are important and must be considered either as variables or, if desired, as a fixed parameter.

In particular the swing-by case becomes very interesting as a spacecraft first goes to one or more flyby planets and then to the destination planet. If the gravitational pull of the flyby planets is utilized effectively in directing the transfer of the satellite towards the destination then the launch velocity goes down. But, as one may expect the time taken to reach the destination planet increases. Hence, there exists a direct conflict between the energy requirements and the total transfer time. Minimizing the total transfer time is important as it helps to avoid any catastrophic event that satellite may encounter while staying in space for too long. Thus, the problem is formulated as a multi-objective optimization problem with objective functions considered as (1) minimization of launch velocity (2) minimization of total time of travel.

The standard orbital mechanics [3] is employed for fixing a trajectory. For computing a transfer from one planet to another, a patched conic model [3] is usually employed. In this model, the motion takes place along a plane. In actual practice, the transfer can involve swing-by planets or can be a direct one. Thus, when a swing-by is to be considered, the spacecraft may have to go through a plane change from one pair of planet transfer to the other. This requires the spacecraft to spend some energy for making a change in its motion from one plane to another. To take care of this additional energy, we add it in the computation of the initial launch velocity. Following subsection briefly describes the Direct and Swing-by Transfers.

2.1 Direct Transfer

Consider a satellite transfer from first to the second planet. This involves knowing the locations of departure and destination planets. Moreover, assume that we fix a transfer time t for reaching second planet from the first one and investigate if such a transfer is possible from the location information of both planets. The so-called Lambert's approach [6] helps us to determine the velocity vectors required at the first (v_1) and the second (v_2) planet in order to materialize such a transfer time. Lambert's approach involves an iterative procedure of adjusting the velocities so that the desired transfer time t is achieved. Thus, for a direct transfer, the departure date and transfer time are the two variables of the optimization problem.

2.2 Swing-by Transfer

In order to have the overall swing-by transfer feasible, the difference between the incoming and outgoing speed at every swing-by planet must be as small as possible [5]. If there are S number of swing-by planets, then in practice, we construct one equality constraint at each swing-by planet:

$$|v_i^+| - |v_i^-| = 0, \quad i = 1, 2, \ldots, S. \tag{1}$$

Above equation says that at i-th swing-by planet, v_i^+ ($+$ mean outgoing from the planet) and v_i^- ($-$ means incoming to the planet) are equal in magnitude. If outgoing v_i^+ and incoming v_i^- do not happen to be in same plane then corresponding energy required to change the plane is accounted and added to the initial launch velocity.

Now, S swing-bys would involve $S+1$ transfer times. To convert the problem in to a root finding problem in $S+1$ transfer times, we introduce another equality constraint as follows:

$$h_1 - h_1^d = 0, \tag{2}$$

where h_1^d is the desired altitude of the first swing-by planet and which is specified. With the help of Lambert's approach $S+1$ transfer times can now be found from above equations. In an overall procedure transfer time values and the altitude of the first swing-by planet are adjusted by using the Newton-Raphson method till the equality constraints are satisfied using a small ϵ value. Thereafter, the original transfer time values are replaced with the ones computed using the Newton-Raphson method. Objective values are then computed for the solution. Variable bounds on transfer times are checked and any violation is assigned as the 'constraint violation' of the solution and the solution is declared infeasible.

2.3 Handling Using NSGA-II

Here we discuss the representation scheme for the decision variables within the NSGA-II framework [4]. A maximum of three swing-by planets is fixed, thereby leaving us with four options: (i) direct flight (no swing-by), (ii) one planet swing-by, (iii) two planet swing-by and (iv) three planet swing-by. A two-bit substring for representing these four options with `00`, `01`, `10` and `11`, respectively, is used. Thereafter, we have three substrings of three bits each. Each three-bit substring represents a swing-by planet (one of the first eight planets of the solar system coded as `000`: Mercury, `001`: Venus, `010`: Earth, etc. Depending on the first two-bit substring dictating the number of swing-by planets we pick the corresponding planets from the string. These $2+3\times3$ or 11-bit strings give us information about which and how many planets are used in the trajectory determination.

The next set of 4 variables are coded as real-valued variables and represent transfer times between departure planet to first swing-by planet, first to second swing-by planet, second to third swing-by planet and third swing-by to arrival planet. Here again, depending on the number of swing-by planets (S) dictated by the first two-bit substring, we consider only the first ($S+1$) transfer times.

A typical NSGA-II solution may look like the following:

`10 000 100 101` 16/6/2005 .15 124 205 580 425

The solution signifies that there are two swing-by planets and they are the first planet (Mercury, 000) and fifth planet (Jupiter, 100) between departure and arrival planets (which need not be represented in NSGA-II, as they are fixed for all solutions). Thus, we ignore the third swing-by planet mentioned in the solution. The next decision variable is the departure date (16 June 2005). This date is actually represented using the Julian day (which is an integer value) . The next variable is height of the altitude above the first swing-by planet as the fraction of the radius of this planet. The next four real-valued values are transfer times and we only pick the first three values as the transfer time between the departure and the first swing-by planet and so on. Once again, the transfer time of 425 is a useless parameter for this solution, since only two swing-bys are considered dictated by the first two-bit substring `10`. Thus, in a case of direct transfers only two variables (the departure date and the first real-parameter value indicating the transfer time). Working out it turns that in case of single swing-by six, two planet swing-by eight and three planet swing-by all ten variables are to be considered.

Along with the evaluation procedure described in the previous section, NSGA-II procedure considers a population of solutions and emphasizes feasible over infeasible solutions, non-dominated solutions over dominated solutions and less-crowded solutions over crowded solutions. We combine the evaluation scheme with NSGA-II and develop a user-friendly software GOSpel(Genetically Optimized Space Launcher).

3. PROOF-OF-PRINCIPLE RESULTS

In the following subsections the correctness of our implementation of trajectory optimization procedure is justified by applying our code on number of known missions, taken from web and literature. Comparison of these results provides authenticity and highlights the fact that GOSpel performs better in many cases. Solving the problem in multi-objective framework provides an insight about the trajectory transfer and also highlights some critical launch opportunities.

3.1 Earth-Venus-Mercury Mission

Table 1: Earth-Venus-Mercury trajectories using $Exhaustive^*$ Search and GOSpel˙

	$Exha.^*$ Search	GOSpel
Earth Departure (mm/dd/yy)	08/05/02	08/05/02
Venus Swing-By (mm/dd/yy	12/05/02	12/05/02
Mercury Arrival (mm/dd/yy	02/13/02	02/07/02
Altitude at Venus (Km)	-938.9	-978.84
Total Time (Days)	192	186
Launch Velocity Km/s	2.79	2.78

Here we consider a Earth-to-Mercury venus mission with a possible swing-by from Venus. This problem was studied by using an exhaustive search procedure [7] for the minimization of launch velocity. For the years 2001 and 2002, the launch possibility and corresponding launch velocity needed for the mission to Mercury via Venus was calculated with a step size of one day for departure. The best solution found is shown in Table 1

To validate our procedure, we use GOSpel during this two-year departure window to find Pareto-optimal solution for the minimization of launch velocity and time of flight. We use the option for using one or no swing-by and the option of an orbital motion to the destination planet. A population of size 200 and a maximum generation of 200 are fixed. The GOSpel software uses the SBX operator with $p_c = 0.9$ and $\eta_c = 10$ and the polynomial mutation operator with $p_m = 1/n$ and $\eta_m = 20$. Figure 1 shows the corresponding frontier. It is interesting to note that there are two disconnected fronts: (i) trajectories with swing-by and (ii) trajectories with direct transfer. For minimum launch vecolity trajectories, it is recommended to use the swing-by from Venus and for minimum time trajectories it is better to go straight to Mercury from Earth. Table 1 shows a closest solution to the exhaustively searched solution for the minimum launch vecolity objective. The GOSpel solution is closer to the previously-reported solution. In fact, since no finite step is used in GOSpel, a better launch-velocity solution than the exhaustive search method (with a step size of one day) is found. The best launch vecolity solution demands a slightly smaller value than the exhaustive search solution. The matching of our results with the exhaustive search solution by an independent study provides confidence to our developed software.

Before we leave this proof-of-principle study, we also plot the departure, swing-by, and arrival dates of all obtained solutions by GOSpel. Figure 1 marks the Julian dates of these trajectories (values marked on the right axis). The following features of trajectores are gathered:

1. All Pareto-optimal missions must start at the same date: 5 October 2002, irrespective of whether the mission involves a swing-by or not.

2. With an increase in launch velocity requirement, the arrival becomes quicker. It seems that if departed from Earth on 5 October 2002, there exist a number of plausible missions trading-off lanuch velocity and travel time.

3.2 Earth-Mars-Venus-Mercury Mission

Next, we apply GOSpel and compare its results with another study performed by a commercial software, Swing-by Calculator (SC) [2] on a mission involving two planet swing-bys: Mars and

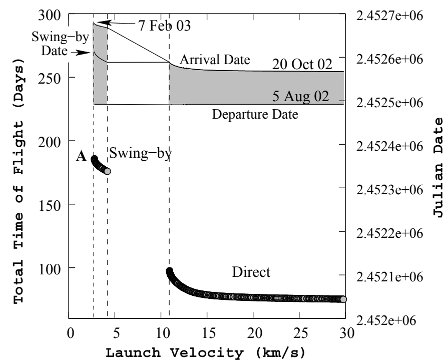

Figure 1: Trade-off solutions for the Earth-Venus-Mercury mission using GOSpel.

Table 2: Earth-Mars-Venus-Mercury trajectories using GOSpel and exhaustive search.

		VSSC	GOSpel
Earth Departure Date	(mm/dd/yy)	08/14/05	08/14/05
Mars swing-by Date	(mm/dd/yy)	10/26/05	10/26/05
Mars: v_∞ incoming	km/s	15.9737	15.9465
Mars: v_∞ outgoing	km/s	15.7722	15.9465
Venus swing-by Date	(mm/dd/yy)	02/01/06	01/31/06
Venus: v_∞ incoming	km/s	8.7439	8.9958
Venus: v_∞ outgoing	km/s	8.9459	8.9958
Mercury Arrival	(mm/dd/yy)	03/31/06	03/31/06
Flight Time	(Days)	229	228.326
Launch Velocity	(km/s)	11.176	11.145

Venus. The destination planet is Mercury and the departure window is kept within 1 Jan 2005 for a year. Table 2 shows the obtained SC result obtained for minimum time of flight. The dates of arrival at Mars and Venus and the corresponding arrival and departure velocities are also shown in the table. The GOSpel solutions are shown in Figure 2 for both objectives. In this case, we allow only two-planet swing-by trajectories to be considered. Thus, direct or one-planet swing-by option is not considered. All solutions found involve two swing-bys, but providing a trade-off between time of flight and energy requirement. The solution on the Pareto-optimal front closest to the SC solution is tabulated in Table 2. The comparison of both solutions again indicates the accuracy of GOSpel procedure. Interestingly, the S solution does not seem to be the minimum-time solution. The figure shows that there exists a solution with a smaller time of flight.

3.3 Cassini Mission

Finally, we consider a mission having three swing-by planets. We found that the Cassini-Huygens mission has four swing-bys [1]. But since our software is limited to a maximum of three swing-by planets, we have considered only the first three out of four swing-by planets in this study. The mission type is set to be a fly-by type at the destination planet. By setting the mission type to fly-by at the destination we expect that our mission's behaviour is similar to transfer which involved the fourth swing-by planet. Thus, the complete mission for the case is departure from Earth, swingby from Venus, another swingby from Venus, third swingby from Earth and

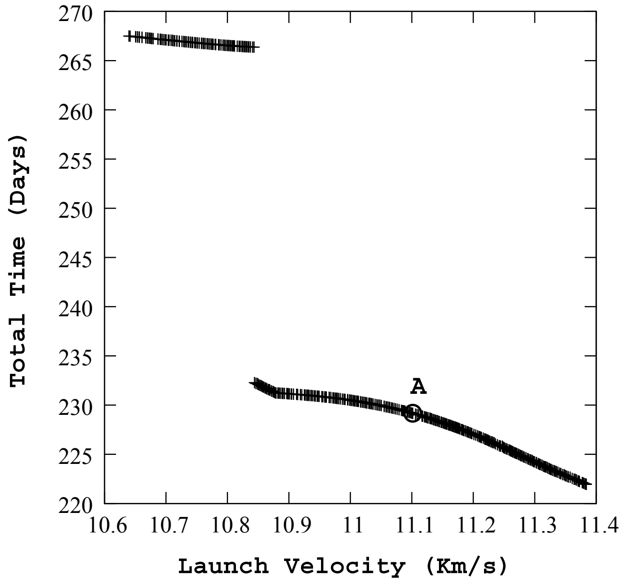

Figure 2: Trade-off solutions for the Earth-Mars-Venus-Mercury mission using GOSpel.

the final arrival to Jupiter. A typical trajectory (taken from [1]), GOSpel and SC solutions are compared in Table 3. From the table, it can be observed that solution found by GOSpel and SC are non-dominated GOSpel solution but time taken are more close to actual mission dates. Time window for departure is considered in a two days span over the actual transfer date. In this case as well GOSpel demonstrates a very good ability of finding trajectory transfers that have been practiced in real-time over past.

Table 3: Earth-Venus-Venus-Earth-Jupiter transfer (part of Cassini mission) using SC and GOSpel.

	Web [1]	GOSpel	SC
Departure Date (mm/dd/yy)	10/15/97	10/15/97	10/29/97
Venus Date (mm/dd/yy)	04/26/98	05/19/98	05/11/98
Earth-to-Venus Time (Days)	194	216.318	194
Venus Date (mm/dd/yy)	06/24/99	06/22/99	06/26/99
Venus-to-Venus Time (Days)	392	399	411
Earth Date (mm/dd/yy)	08/18/99	08/17/99	08/18/99
Venus-to-Earth Time (Days)	55	55.940	53
Jupiter Arrival Date (mm/dd/yy)	12/30/00	12/22/00	02/02/01
Earth-to-Jupiter Time (Days)	500	493.46	534
Total Flight Time (Days)	1168	1164.72	1192
Launch Velocity (km/s)	NA	6.805	4.46

4. CONCLUSIONS

In this paper, we have discussed the development of a multi-objective optimization software (GOSpel) for finding various optimal interplanetary trajectories between any two planets for a dual minimization of travel time and launch velocity. The software is capable of considering a maximum of three swing-bys of intermediate planets to assist in reducing the fuel comsumption. The use Pareto-optimality concept and genetic algorithms has demonstrated that the proposed approach can be used to find a set of trade-off solutions which match with the existing solutions of known missions. Thereafter, the developed code is applied to a number of complex case studies and interesting solutions have been obtained. This paper has amply shown the usefulness and flexibility of such a code for real-time application of EMO for interplanetary trajectory optimization.

5. ACKNOWLEDGMENTS

The author greatly acknowledges the most sincere guidance of his supervisor Dr. Kalyanmoy Deb without which this work would not have materialized. Author also appreciates the effort of Mr. Ganeesh Neema for playing very important role in problem formulation. Also, gratitude is expressed to all members of KanGal whose support has been precious. The project team also appreciate support from ISRO-IITK Cell for executing this study. All time inspiring support from family was most precious.

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