Speed-up Techniques for Solving Large-scale bTSP with the Two-Phase Pareto Local Search

Thibaut Lust
Laboratory of Mathematics & Operational Research, Polytechnic Faculty of Mons
9, rue de Houdain
7000 Mons, Belgium
thibaut.lust@fpms.ac.be

ABSTRACT

We first present a method, called Two-Phase Pareto Local Search, to find a good approximation of the efficient set of the biobjective traveling salesman problem. In the first phase of the method, an initial population composed of an approximation of the extreme supported efficient solutions is generated. We use as second phase a Pareto Local Search method applied to all solutions of the initial population. We show that using the combination of these two techniques: good initial population generation and Pareto Local Search gives good results, without numerical parameters. As the computational time of the second phase grows exponentially according to the instances size, speed-up techniques are used to considerably reduce the computational time of the second phase. It makes it possible to find good approximation of the efficient set of large-scale biobjective traveling salesman problems, in a reasonable resolution time.

Categories and Subject Descriptors

F.2 [**Analysis of Algorithms and Problem Complexity**]: Miscellaneous

General Terms

Algorithms, Performance

Keywords

Multiobjective combinatorial optimization, tsp, local search, speed-up techniques

1. INTRODUCTION

Considering more than one objective in combinatorial optimization considerably increases the complexity of resolution, even if the multiobjective problems are derived from single-objective problems solvable in polynomial time. Thereby, during the last two decades, many papers have been published on the adaptation of metaheuristics to multiobjective problems [5]. The majority of the developed methods follows the idea of one (hybrid) metaheuristic. As at the origin the metaheuristics were dedicated to single-objective optimization, components suitable for multiobjective optimization have been integrated. In consequence, many new parameters have to be tuned for getting good performances.

Our approach to find approximations of multiobjective problems is rather different. Initially, we benefit to the maximum from very efficient heuristics developed for the resolution of the corresponding single-objective problems. In a second time, we use the adaptation of one of the most simple metaheuristic: the hill-climbing method. This approach implies that no new numerical parameters are introduced.

At final, we obtain a simple method, called Two-Phase Pareto Local Search (2PPLS), with no numerical parameters and with a natural stop criterion. We apply the method to the biobjective traveling salesman problem, and we obtain better results on several indicators than state-of-the-art algorithms, quite complicated methods demanding sometimes many parameters.

A weak point of the method is the resolution time that becomes high when larger instances are tried to be solved. Therefore, we show how to adapt the traditional speed-up techniques developed for the single-objective TSP to reduce the resolution time of the 2PPLS method.

2. THE BIOBJECTIVE TRAVELING SALESMAN PROBLEM

Given a set $\{v_1, v_2, \cdots, v_N\}$ of cities and two costs $c_1(v_i, v_j)$ and $c_2(v_i, v_j)$ between each pair of distinct cities $\{v_i, v_j\}$ (with $i \neq j$), the biobjective traveling salesman problem (bTSP) consists of finding a solution, that is an order π of the cities, so as to minimize the following costs ($k = 1, 2$):

$$\text{``}\min\text{''} z_k(\pi) = \sum_{i=1}^{N-1} c_k(v_{\pi(i)}, v_{\pi(i+1)}) + c_k(v_{\pi(N)}, v_{\pi(1)})$$

Hence, two values are associated to an order π. We are interested here only in the symmetric biobjective traveling salesman problem (bTSP), that is $c_k(v_i, v_j) = c_k(v_j, v_i)$ for $1 \leq i, j \leq N$.

In this paper, we use biobjective instances of size going from 100 to 1000. The instances with less than or equal to 200 cities have been generated on the basis of single-objective TSP instances of the TSPLIB library [7]. For the instances of at least 300 cities, we have generated ourselves the bTSP instances, by randomly generating coordinates.

3. TWO-PHASE PARETO LOCAL SEARCH

The spirit of the two phases of the Two-Phase Pareto Local Search is similar to that of the exact Two-Phase method developed by Ulungu and Teghem [8], but here, approximation methods are used in both phases. The two phases of the method are as follows:

1. Phase 1: Find a good approximation of the supported efficient solutions. These solutions can be generated by resolution of weighted sum single-objective problems obtained by applying a linear aggregation of the objectives. We limit ourselves to find a good approximation of a minimal complete set of the extreme supported efficient solutions. In this aim, we have heuristically adapted the method of Aneja and Nair [1], initially proposed for the resolution of a biobjective transportation problem. The method consists in generating all the weight sets which make it possible to obtain a minimal complete set of extreme supported efficient solutions of a biobjective problem (non-extreme supported efficient solutions and equivalent solutions can however be generated).

2. Phase 2: Find non-supported efficient solutions located between the supported efficient solutions. In this phase, we use the Pareto Local Search (PLS) method, used and developed by different authors [2, 3, 6].

The results of the comparison of the Two-Phase Pareto Local Search (2PPLS) method with state-of-the-art algorithms show that the 2PPLS method is better on several indicators.

Concerning the resolution time of 2PPLS, we remark that the resolution time of the first phase increases more or less linearly according to the instance size. On the other hand, the resolution time of the second phase, the Pareto Local Search, strongly increases. Indeed, in the second phase, we totally explore the neighborhood of every solution of a population, by making 2-exchange movements. The complexity of such neighborhood is in $O(n^2)$. As a result, solving instances of more than 500 cities with the 2PPLS method without speed-up techniques is practically impossible. Effectively, we did not manage to solve the instances of 750 and 1000 cites in a reasonable time (for the 500 cities instance, the second phase already takes more than 6000S).

Many speed-up techniques have been developed for the single-objective TSP [4], but as our knowledge, none of these techniques have been adapted to the resolution of the bTSP (excluding biobjective instances resolved by a method using aggregation functions to transform the biobjective problem into several single-objective problems).

Hence, we present at the next section speed-up techniques for solving the bTSP with the 2PPLS method, to reduce the resolution time of the second phase.

4. SPEED-UP TECHNIQUES

We propose different speed-up techniques for solving large-scale bTSP instances with 2PPLS: neighbor lists based on k-nearest neighbors, on data dominance relations and on the edges used by the solutions generated after the first phase of 2PPLS. We also propose an adaptation of the "don't-look bits" technique for the biobjective TSP.

None of these techniques guarantee such good results as found with the 2PPLS method with a complete exploration of the neighborhood. But we show that the results remain practically the same with a consequent gain of time. Considering only the edges used by the solutions generated after the first phase, for the second phase, is already a good compromise between performances and resolution time.

We also give by this work state-of-the-art results for biobjective instances of the TSP with more than 200 cities, until 1000 cities, which is an instance size that has never been tackled.

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