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# Frame Design Synthesis Using Implicit Redundant Genetic Algorithm

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## Abstract

Synthesis design solutions for an unstructured, multi-objective problem domain are evolved using the implicit redundant representation genetic algorithm (IRR GA). The IRR GA uses redundancy to represent a variable number of location independent design parameters. Using the IRR GA in tandem with an unstructured definition of the problem domain allows the representation and evaluation of diverse structural topologies and geometries. Details of the IRR GA design parameter encoding and the unstructured formulation of the frame synthesis design problem are discussed along with the GA fitness and penalty functions applied. Novel frame designs generated by the IRR GA synthesis design method, which compare favorably with traditional frame design solutions obtained by trial and error, are presented.

## 1 INTRODUCTION

Performing synthesis during conceptual design provides substantial cost savings by selecting the structural topology and geometry of the design, in addition to selecting the member sizes. Traditional optimization methods cannot effectively synthesize design solutions that have diverse structural topologies and geometries. In the past, shape optimization methods were used to refine the member section properties of structures having a fixed topology and geometry as a final design stage to reduce cost by reducing the volume of material used. The research presented by this paper focuses on topology and geometry optimization of the structure, in addition to shape optimization, by supporting the synthesis of design alternatives during conceptual design. The cost benefits of design changes made during the conceptual design stage are greater than any design changes identified during the final design stage (Reich & Fenves, 1995).

In structural design, topology optimization defines the number of joints in the structure, the joint support locations, and the number of members connected to each joint. Geometry optimization defines the length of the members and the loca-

tion of joints within the problem domain. The x, y, or z coordinates of the joints must be designated as design variables to optimize the geometry of the structure. Topology optimization requires the ability to add and remove members and joints from the structure, either heuristically or implicitly.

Researchers have applied genetic algorithms (GA) to structural truss optimization problems, including the optimization of trusses with fixed topology and fixed geometry (Adeli & Cheng, 1993; Yang & Soh, 1997); the optimization of trusses with fixed topology and variable geometry (Wu & Chow, 1995); and the optimization of trusses with variable topology and geometry (Rajan, 1995; Rajeev & Krishnamoorthy, 1997). These GA truss topology and geometry optimization methods, including those based on the ground structure approach (Hajela & Lee, 1995; Rajan, 1995), are not directly transferable to frame optimization problems due to the non-linear interactions existing between the member properties and the member stresses and the use of heuristic rules for adding or deleting members. Frame designs having diverse topologies and geometries can satisfy the design objectives equally well and obtaining a good design solution requires a trial and error process. Several researchers have used GAs to optimize the members sizes of frame structures with fixed geometry and topology (Camp, et al., 1998) and to provide limited geometry optimization of frame structures (Grierson & Park, 1996; Jenkins, 1997). The synthesis method implemented using the IRR GA and an unstructured problem domain formulation discussed by this paper provides both structural frame topology and geometry optimization.

## 2 IMPLICIT REDUNDANT REPRESENTATION

In order to provide an evolutionary based method capable of synthesizing design alternatives, a more flexible GA representation is required that is capable of encoding a variable number of design variables, providing location independent design variables, and allowing self-organization of the linkage of the encoded design variables. The implicit redundant representation (IRR) provides a mechanism that allows essential and redundant sections of a string to interact dynamically by using a string length that is longer than the length required to encode only the parameter values (Raich & Ghaboussi, 1997). The specific location of each encoded pa-

parameter value, which is called a gene instance, is not designated explicitly by the IRR. Instead, each gene instance is allowed to drift within the length of the string as shown in Figure 1. Each gene instance in an IRR string consists of two parts: a pre-selected Gene Locator (GL) pattern identifying the location of the gene instance in the string and a specified number of useful bits of the gene instance that encode the parameter values. All population individuals have the same string length and each individual in the population represents one complete solution. To decode the parameter values from the IRR string, the string is parsed until a GL pattern is found indicating a gene instance. The parameter values are encoded using binary or real numbers similar to other GAs.

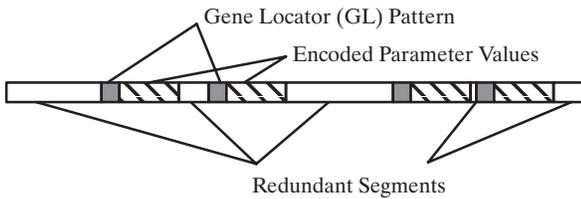


Figure 1: Generic IRR GA genotype.

The portions of the string that are not part of a gene instance contain redundant material. Incorporating the use of redundant, or non-coding segments, has been researched previously (Levenick, 1991; Wu & Lindsay, 1996; Raich & Ghaboussi, 1997). Each redundant segment consists of a variable number of bits that separate the gene instances in the string. The use of redundancy provides several benefits to the evolutionary process: redundant segments protect existing parameters from the disruption of crossover and mutation and new gene instances may be designated within previously redundant segments by the actions of crossover or mutation in future generations (Raich & Ghaboussi, 1997).

In addition, the designer is not required to specify the number of parameter values to be represented by the IRR GA. Instead, the number of variables encoded changes dynamically from generation to generation. No external constraints are required to process over or underspecified strings, since the IRR GA strings are the same length.

### 3 UNSTRUCTURED PROBLEM FORMULATION

The synthesis of design alternatives is supported in this research by defining an unstructured problem domain that does not have explicit bounds placed on the design parameters modeled. Therefore, design solutions can be generated and compared that have diverse topologies and geometries. Synthesis of design alternatives has two principles driving it: providing partial optimality of design (in some sense the best design) and ensuring feasibility of design. Synthesis alternatives are found in the search space bounded by the space of all possible design alternatives, which is infinite and ill-defined, and the space of mathematical programming design alternatives, which is very small and well-defined.

The level of unstructuredness of the design domain is altered by placing constraints on the values of the design variables within the problem domain. A tradeoff occurs in the process of determining the level of unstructuredness that is beneficial to the synthesis process. Removing constraints allows for a more diverse set of synthesis alternatives to be explored by expanding the search space. The increased exploration for design alternatives, however, has a high computational cost attached. Limiting the size of the search space by constraining the values of specific design variables results in a limited exploration, with the cost being the exclusion of beneficial design alternatives from consideration.

A design problem with a predefined topology and geometry has a fixed number of variables, a bounded search space, and a single, static fitness landscape, which may be multi-modal. In unstructured problem domains, however, there is no assurance that the fitness landscape searched remains static. Instead, each distinct topology and geometry considered will have a fitness landscape defined in a distinct dimensional search space. If the topology or geometry changes, then a new fitness landscape will be defined. The search for synthesis design solutions in an unstructured problem domain is performed over a non-stationary fitness landscape. Unstructured problems, therefore, can be categorized as highly deceptive problems (Goldberg, 1989).

### 4 PROBLEM STATEMENT

The design problem selected for this paper is the synthesis of a plane frame structure with a maximum total structure width of 60'-0" and a maximum structure height of 36'-0" (three floors). The unstructured frame problem domain is defined as shown in Figure 2. and is defined by: dimensional bounds placed on the maximum structure width and height and the statement of the location of planes of possible applied loading and possible support placement. The actual loading applied to the frame structure is a function of the number of stories and bays defined and varies for each individual frame synthesis alternative. The fixed design parameters are the magnitudes of the dead load, live load and wind load and the designation of pinned support nodes. All other required design information, including the number and location of structural nodes and members, member properties, support information, member connectivity, number of stories, and number and size of bays, are design variables.

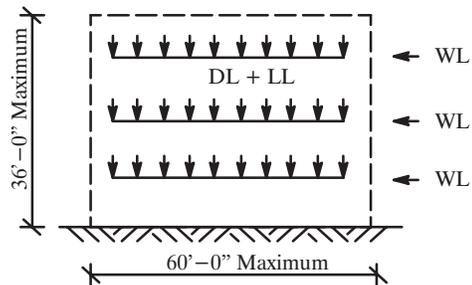


Figure 2: Model of unstructured problem domain for the frame synthesis design problem.

#### 4.1 IRR GA FRAME SYNTHESIS GENE INSTANCE

Assembling a frame design solution within the unstructured problem domain requires knowledge about the number of members, the member areas, and the member locations in the structure as defined by the nodal coordinates. The topology and geometry of the structure is specified through the designation of member and nodal information using a design grammar. The process of defining the required design grammar for the frame design problem is simplified because the grammar is explicit in the genotype/phenotype relationship provided by the IRR GA representation itself.

The design information required to model a frame member is encoded in a single gene instance identified by the GL pattern [1 1 1] in the order shown in Figure 3: the x-coordinate of node 1 (X1); the y-coordinate of node 1 (Y1); the x-coordinate of node 2 (X2); the y-coordinate of node 2 (Y2); the depth of the non-horizontal member (Depth 1); the depth of any horizontal member connected to the right of node 1 (Depth 2); and the depth of any horizontal member connected to the right of node 2 (Depth 3). This design information defines the non-horizontal member coordinates, nodal incidences, and member depths as shown in Figure 4. Members decoded from the IRR genotype having the same y-coordinates for both nodes, which designates a horizontal member, are ignored during the assembly of the non-horizontal members. The total number of frame members (gene instances) encoded in each IRR GA genotype is implicitly constrained by the fitness and penalty functions and will vary among the individuals in the current population. Representing different structural topologies and geometries is achieved by encoding different numbers of location independent design variables in the IRR GA string.

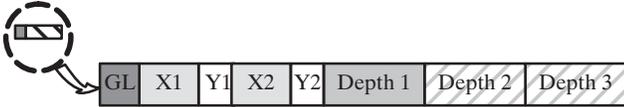


Figure 3: IRR GA gene instance for the unstructured frame problem domain.

The design variable value ranges are set by the number of binary bits used to encode each variable. The nodal x-coordinates, X1 and X2, are encoded as 6-bit binary numbers that are mapped by the following function:  $(X1 - 31.0) * 12.0$ , which encodes a value range of (-372.0, 384.0) with a unit of inches. The y-coordinates, Y1 and Y2, are encoded as 2-bit binary numbers. Each of the four encoded binary values corresponds to a floor level of 0, 1, 2, or 3. All three member depths are 3-bit binary numbers that encode 8 discrete member depths {5, 10, 15, 20, 25, 30, 40, 50} with a unit of inches. All of the structural frame members are defined as steel tube sections having a fixed width and thickness and a variable decoded depth. The member area and the section modulus are calculated based on the member depth decoded.

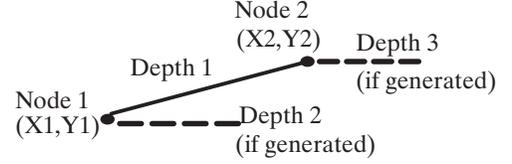


Figure 4: Design variables for a single member in the unstructured frame problem encoded in one gene instance.

The two horizontal member depths decoded from the gene instance for each non-horizontal member are used when a horizontal member is generated. The horizontal members are generated between each pair of adjacent nodes defined on the same floor level from the non-horizontal member information decoded. The depth of horizontal member is provided by the value of the horizontal depth (Depth 2 or Depth 3) decoded for the designated starting node of the horizontal member as shown in Figure 4. Assembling a complete frame structure consists of defining the non-horizontal member locations using the nodal coordinates decoded from the IRR genotype and generating the horizontal members by connecting the nodal coordinates defined along each level.

Three repair strategies were applied to the complete frame structures as required: assigning a minimal fitness to frames that have less than two supports to prevent unstable structures from being analyzed; replacing nodes that are closer than 5'-0" with a single node to reduce the automatic generation of very short members; and removing single nodes that occur within the structure that do not carry any loading.

#### 4.2 FRAME FITNESS AND PENALTY FUNCTIONS

Typically, a frame design problem has a single objective: provide minimum weight subject to the satisfaction of flexural strength requirements and deflection requirements. Satisfying this objective using the unstructured frame problem domain, however, results in the evolution of minimal structures represented by two member frames that carry no loading. Therefore, a second objective is required: maximize the total floor space provided by the frame. The non-penalized GA fitness functions that optimize the volume (minimum weight),  $F_V$ , and floor area,  $F_F$ , objective functions can be stated for the frame synthesis design problem :

$$F_V = \left[ \frac{\left[ C_V - \sum_{i=1}^m \rho A_i l_i \right]}{C_V} \right]^{\alpha_V} \quad F_F = \left[ \frac{\sum_{j=1}^{m_h} h_j(x_j)}{L_H} \right]^{\alpha_F} \quad (1)$$

where  $m$  is the total number of members;  $m_h$  is the number of horizontal members;  $C_V$  is a selected scalar value that is larger than the maximum expected volume;  $L_H$  is the maximum total floor space provided by the dimensional bounds placed on the problem domain; and  $\alpha_V$  and  $\alpha_F$  are selected exponential power terms.

A stress penalty function,  $P_S$ , is used to reduce the GA fitness of frame design solutions that violate the maximum stress criteria of the design code:

$$P_S = \left[ \frac{C_S - \prod_{j=1}^m \text{Int}(M_j, M_{j,all}, P_j, P_{j,all})}{C_S} \right]^{\alpha_S} \quad (2)$$

where  $\text{Int}()$  is the interaction ratio defined by the LRFD code,  $M_j$  is the design moment in member  $j$ ;  $M_{j,all}$  is the allowable moment in member  $j$ ;  $P_j$  is the design axial force in member  $j$ ;  $P_{j,all}$  is the allowable axial force in member  $j$ ;  $\alpha_S$  is a selected exponential power term; and  $C_S$  is a selected scalar value that is larger than the maximum stress interaction penalty.

The evolved IRR GA frame design solutions must also satisfy serviceability criteria that require the horizontal deflection of the structure to satisfy the NEHRP allowable inter-story drift limits and restricts the vertical deflection of the structural member to a deflection of less than  $l/360$  across the member. The penalty functions,  $P_{HD}$  and  $P_{VD}$ , used to reduce the fitness of design solutions that have excessive horizontal and vertical deflections can be stated (with an additional subscript of  $H$  for horizontal and  $V$  for vertical deflection):

$$P_D = \left[ \frac{C_D - \prod_{l=1}^n \left( 1.0 + \frac{\Delta_l}{\Delta_{max}} \right)}{C_D} \right]^{\alpha_D} \quad (3)$$

where  $n$  is the number of nodes considered for horizontal or vertical deflection;  $\Delta_l$  is the horizontal or vertical deflection of node  $l$  exceeding the set limit;  $\Delta_{max}$  is the maximum limit on horizontal or vertical deflection for all nodes;  $\alpha$  is a selected exponential power term, and  $C$  is a selected scalar value that is larger than the maximum horizontal or vertical deflection penalty.

Aesthetics are introduced into the synthesis search process by promoting the symmetric placement of structural members and nodes, while still allowing the consideration of nonsymmetrical member and node placement. Penalties for nonsymmetrical members and nodes in the structure are calculated using a 2'-0" tolerance. The nodal and member symmetry penalty functions,  $P_{SN}$  and  $P_{SM}$ , can be stated that penalize the design solution:

$$P_{SN} = \left[ \frac{1.0}{\sum_{k=1}^n (\text{num\_Sym} * 0.2)} \right]^{\alpha_{SN}} \quad P_{SM} = \left[ \frac{1.0}{\sum_{k=1}^m \text{sym}(k,j)} \right]^{\alpha_{SM}} \quad (4)$$

where  $\text{num\_Sym}$  is the number of non-symmetrical nodes;  $\alpha_{SN}$  and  $\alpha_{SM}$  are selected exponential power terms; and

$\text{sym}(k,j)$  is 0 if members  $k$  and  $j$  are symmetric or 1 if members  $k$  and  $j$  are not symmetric.

Applying the LRFD load combinations to structures that are potentially nonsymmetrical requires the analysis of four loading cases: two load cases for Dead Load + Live Load on alternating spans and two load cases for Dead Load + Wing Load from two directions. Three of the penalty functions, stress ( $P_S$ ), horizontal deflection ( $P_{HD}$ ), and vertical deflection ( $P_{VD}$ ), must be evaluated for each of the four code specified loading conditions applied to the structure to determine the total penalty function. The calculation of the stress and deflection penalties requires a separate structural analysis for each individual in the IRR GA population. Using an unstructured formulation for the plane frame design problem creates a difficulty in applying the gravity and wind loading to the structure. The loading cannot be applied to a fixed set of members or nodes, since the same members and nodes are not always present due to variable geometry and topology. Instead, the loading applied depends on the topology and geometry of the each structure. Gravity load is applied uniformly along the horizontal members defined at each floor level. The alternating spans are defined by the location of the nodes along each floor level and do not necessarily relate to equal spans. The wind load is applied to the exterior nodes defined at each floor level depending on the direction of the wind. If a floor level is not defined at a specific level, the wind load is transferred to the floors above and below the non-existent floor level.

A product composite penalty term,  $P_{TOT}$ , that magnifies the differences existing among the individual penalty terms defined in Equations 2 to 4 was defined:

$$P_{TOT} = \sum_{k=1}^l P_S^k * \sum_{k=1}^h P_{HD}^k * \sum_{k=1}^j P_{VD}^k * P_{SN} * P_{SM} \quad (5)$$

where  $l$  is the number of loading cases analyzed;  $h$  is the number of load cases analyzed for horizontal deflection; and  $j$  is the number of load cases analyzed for vertical deflection.

Experimental results for the frame synthesis design problem presented in this paper were obtained using a product composite fitness function that is composed of two fitness terms and ten penalty terms:

$$\max F [x] = F_V * F_F * P_{TOT} \quad (6)$$

The values of the scalar terms stated in the fitness and penalty functions defined by Equations 1 to 4 are provided:  $C_V = 600.0$ ;  $C_S, C_{VD}, C_{HD} = 2000.0$ ;  $\alpha_v, \alpha_F = 1.0$ ;  $\alpha_{VD}, \alpha_{HD} = 4.0$ ;  $\alpha_{SN}, \alpha_{SM} = 0.1$ ; and  $L_H = 2268.0$ .

### 4.3 SELECTION OF IRR GA STRING LENGTH

The selection of the appropriate level of redundancy is an important design consideration (Raich & Ghaboussi, 1997). The GL pattern selected affects the number of gene instances initialized in the randomly generated population of strings. This effect is directly related to the probability of an occurrence of the GL pattern within the designated string length.

For a IRR GA string length of 600 bits and a total gene instance length of 22 bits, an average of 16 members (gene instances) are randomly initialized in each individual (genotype). Starting with an overspecified string provides more diversity initially to the solution process. The overspecified string protects the population from premature convergence by reducing the average stress and deflection penalties during early generations, which lowers the severity of the penalties.

#### 4.4 GENETIC CONTROL OPERATORS USED

The search space for the frame synthesis design problem includes multiple, equally optimal solutions. To ensure that the population did not converge to a single optimum, fitness sharing was used to distribute the population among multiple solutions with only a few individuals maintained in the vicinity of each solution in the search space (Goldberg & Richardson, 1987). A niche count,  $m_i$ , was used to reduce the fitness of similar individuals. The sharing function applied was the same as defined by Goldberg (1989) with a similarity measure,  $\sigma_s$ , of 0.05 to control the size of the niche. An Euclidean distance measure was calculated to relate the similarity between the satisfaction of the individual objective and penalty terms for all individuals in the population. The fitness of each individual was reduced based on the number of similar individuals in the current population as defined by the niche count,  $m_i$ . Tournament selection was performed using the modified fitness values to determine the next generation population. A tournament group of  $n$  individuals was selected for competition. The individual with the highest fitness in the tournament group was selected to be the winner of the tournament. To ensure that the fittest individual in the current population was not removed because of a low selection pressure or destroyed because of the disruption of crossover or mutation, an elitist strategy was used. The fittest individual in the current population was copied to the next generation bypassing any genetic manipulation.

To increase the number of string segments recombined and to reduce the size of each of the string segments exchanged, multiple point crossover was used. A random, normal distribution was used to select the number of crossover sites using a mean of 10 crossovers and a standard deviation of two. A crossover rate of 1.0 was used. Single bit mutation was applied to the population using a mutation rate of either 0.0025 or 0.0033.

### 5 FRAME SYNTHESIS DESIGN RESULTS

Experimental trials were performed using the IRR GA to model the fully unstructured plane frame problem domain defined in Figure 2. Three frame synthesis design solutions evolved by the IRR GA after 1500 generations using a population size of 200, a string length of 800 bits, and a tournament size of 10 are presented in Figure 5. The IRR GA trials were randomly initialized using different random seeds. The product composite fitness function defined by Equation 6 was used for these trials. Each of the IRR GA frame synthesis design trials converged to design solutions that had three stories

and the maximum floor space allowed by the domain boundaries. The beneficial influence of the member symmetry penalty on the evolution of design features and complete frame design solutions is apparent. Incorporating symmetry into the design process by penalizing unsymmetrical solutions allowed the evaluation of single members before promoting the addition of symmetrical members to the structure. To support member symmetry, the IRR GA genotype must maintain two separate gene instances, which correspond to the pair of symmetrical members, within the genotype. The self-organization of the location independent gene instances along the IRR GA genotype helps to protect these pairs of gene instances from the disruption of crossover and mutation. If this flexibility of encoding is not provided, representing structures having multiple pairs of symmetrical members is difficult.

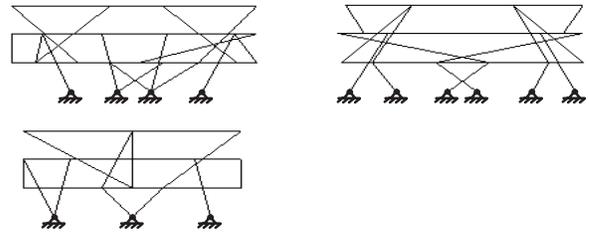


Figure 5: IRR GA frame design solutions represented by the fittest population individual after 1500 generations.

Additional IRR GA frame synthesis design trials are presented using a less unstructured problem formulation. The nodal  $x$ -coordinates ( $X1$  and  $X2$ ) encoded in the genotype gene instances were restricted to a 10'-0" spacing along each floor, instead of the 1'-0" spacing used in the previous trials by using 3-bit binary encodings. Multiple, randomly initialized IRR GA trials were performed using the product composite fitness function defined by Equation 6, a population size of either 100 or 200, a string length of 600, and a tournament size of 5. The reduced population size and string length resulted from constraining the size of the search space by reducing the number of possible  $x$ -coordinate locations assigned. The frame synthesis design solutions obtained after 500 generations for four IRR GA trials are shown in Figure 6. The evolved synthesis design solutions satisfied the symmetry penalties to a greater extent than the trials performed without placing a restriction of the  $x$ -coordinates of the nodes.

The frame synthesis design solutions shown in Figure 6 provide good complete frame design solutions that incorporate inclined columns to aid in resisting wind loading, tension members carrying gravity loading, and stiff, triangular substructures. A striking feature of these designs was the specification of separate load carrying systems for the individual floors. For the design solution shown in the bottom, right-hand corner of Figure 6, the second and third floor loadings are carried to the foundation through an arch structural system. The first floor loading is carried on additional vertical, one-story columns.

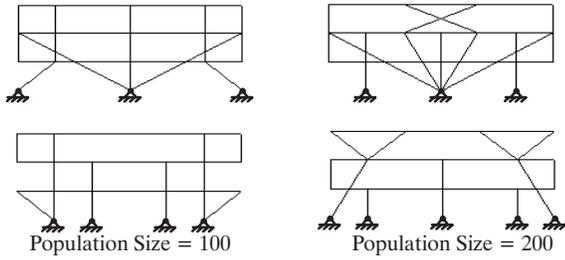


Figure 6: IRR GA frame design solutions represented by the fittest population individual after 500 generations.

### 5.1 EVOLUTION OF FRAME SYNTHESIS DESIGN SOLUTIONS

The IRR GA evolutionary process starts with a population of randomly initialized individuals. During each generation, a new population is selected based on fitness, with those individuals having a higher fitness having a greater probability of being selected. Crossover and mutation are applied to the selected individuals to create new individuals that retain the beneficial characteristics of their predecessors. The process of evolving frame synthesis design solutions can be investigated by examining the features of the fittest individual in the IRR GA population at specific generations. Figure 7 presents an overview of the evolutionary search process for one of the IRR GA frame synthesis design trials shown in Figure 6.

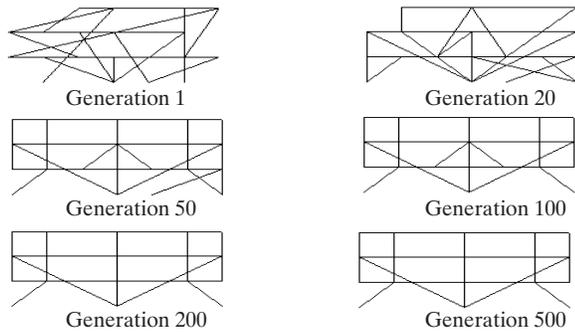


Figure 7: Example of the evolution of the best IRR GA design solution at each generation.

After one generation, the frame design solution represented by the IRR GA population individual is not random since one tournament selection has been performed. Although three stories are defined by the design solution, the floor space provided by each story does not extend to the domain boundaries. The frame design solution also has more members than are required, which prevents assigning extremely high stress and deflection penalties to a high percentage of the population. The best frame design solution at 20 generations included several design features that were similar to the features found in the final design solution. After 50 generations, the influence of the floor space objective and the nodal symmetry penalty begins to appear. The floor space provided is extended

towards the domain boundaries and the nodal coordinates are placed in nearly symmetrical positions. The synthesis of the topology and geometry of the design solution continued until 200 generations were performed. Shape optimization of the member depths was performed after 200 generations on the synthesized topology and geometry. The best frame design solution at 500 generations is symmetric and optimizes the floor space and the volume objectives well. The design solution also satisfies the stress, deflection, and symmetry penalties for each of the four applied loading configurations.

### 5.2 DIVERSITY OF IRR GA POPULATION

The diversity of the population during evolution can be investigated by comparing the maximum fitness obtained by the population at each generation with the average fitness of the population. Figure 8 presents a plot of both the maximum and average fitness for the IRR GA trial shown and presented in Figure 7. During early generations, the IRR GA is synthesizing the topology and geometry of the design solutions. For the remaining generations (after generation 200 for the trial shown in Figure 8), the IRR GA performs member size optimization on the best fixed structural topology and geometry that is evolved. The disruptive effect of crossover and mutation on the IRR GA genotype, however, makes the size optimization process difficult. The flexibility provided to encode variable topologies and geometries during synthesis does not provide the best representation for performing size optimization. The average population fitness did not converge to the maximum fitness after synthesis was completed. Convergence of the population indicates that the population individuals are becoming similar and are representing similar design solutions. Instead, the population diversity was maintained throughout the entire evolutionary process of synthesis and optimization for the IRR GA trial.

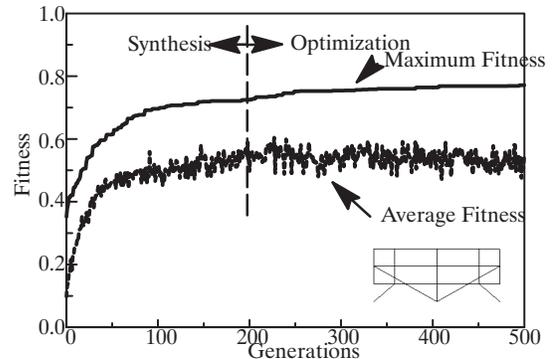


Figure 8: Maximum and average fitness of the IRR GA population over 500 generations for a single trial with a 10'–0" restriction on the x-coordinate spacing.

### 5.3 COMPARISON OF IRR GA AND STANDARD FRAME DESIGN SOLUTIONS

The quality, or the optimality, of the IRR GA frame design solutions cannot be determined directly based on a comparison with known optimal frame design solutions. The frame design problem has numerous optimal solutions that each sat-

isfy the constraints and optimize the objectives equally well using different structural configurations of member sizes, topologies, and geometries. Three frame design solutions were determined using a trial and error design process using standard frame topologies and geometries. The standard frame design solutions used only vertical columns and rectangular bays and are shown graphically in Table 1 as frame design solutions I, II, and III. Standard frame design I has three, 12'-0" stories and three, 20'-0" bays. Standard frame design II has three, 12'-0" stories and two, 30'-0" bays. Standard frame design III has three, 12'-0" stories and three bays, but with a 10'-0" wide interior bay and two, 25'-0" wide exterior bays.

Table 1 presents the structural performance of the three standard frame design solutions. The categories used for comparison are the total volume of the structure, the maximum horizontal deflection, the maximum vertical deflection, the average stress ratio in the horizontal members, and the average stress ratio in the non-horizontal members. Two values of average stress ratio are provided: the stress induced by gravity loading and the stress induced by wind loading, which is indicated by italics. Standard frame design solution I provided the lowest volume of the three design solutions. All three design solutions, I, II, and III, provided approximately the same level of structural deflection and relatively low levels of average stress ratios under both gravity and wind loading.

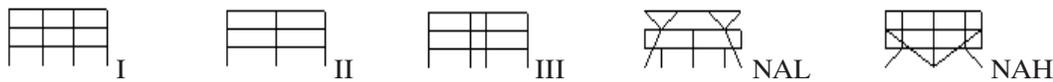
Two IRR GA frame synthesis design solutions are selected from the previous results for comparison with the standard frame design solutions. IRR GA trials (NAL and NAH) shown in Table 1 restricted the nodal x-coordinates to 10'-0" spacings. The IRR GA frame design solutions presented pro-

vide competitive solutions when compared with the standard frame design solutions generated using trial and error. Both IRR GA trials have volumes that compare favorably with the standard frame design solution volumes. The average horizontal and non-horizontal member stress ratios for the IRR GA synthesis solutions are lower than those maintained by the standard design solutions.

The IRR GA synthesis design method evolved solutions that have a symmetric topology and geometry, but that do not necessarily have symmetric member sizes. Both IRR GA trials presented in Table 1 were evaluated after modifications were performed to provide symmetric member sizes and to also reduce any excessive deflection of specific nodes. The excessive deflections that occurred in the IRR GA frame design solutions were located at the nodes of members that cantilever from the main supporting members. The NAL (Modified) trial provided a very competitive design solution. The other IRR GA trial (NAH) reduced the excessive deflections at the expense of increasing the structural volume.

In addition, one IRR GA trial was selected and changes were made to the evolved structural member sizes to increase the average stress ratios in the horizontal and non-horizontal members, resulting in the best frame design solution of all of the alternatives examined. The IRR GA trial NAL (Ratio) design solution provided the structure with the lowest volume (90.665), while increasing the average stress ratios to levels closer to those provided by the standard frame design solutions examined. The low stress ratios maintained by the IRR GA design solution could be addressed in future trials by penalizing the design solutions for under-stress of the members in addition to the current penalty for over-stress of the members.

Table 1: Evolutionary Frame Solutions vs. Trial and Error Frame Solutions



| Structure                              | Volume (ft <sup>3</sup> ) | $\Delta_{\max}$ Horizontal (in.) | $\Delta_{\max}$ Vertical (in.) | Horizontal Member Stress Ratio | Vertical Member Stress Ratio |
|--|---------------------------|----------------------------------|--------------------------------|--------------------------------|------------------------------|
| Solutions Generated by Trial and Error |                           |                                  |                                |                                |                              |
| I                                      | 99.666                    | 0.5241                           | 0.25                           | 0.527916/0.295562              | 0.290942/0.185533            |
| II                                     | 102.666                   | 0.5585                           | 0.25                           | 0.546607/0.418753              | 0.416128/0.257392            |
| III                                    | 105.499                   | 0.5704                           | 0.25                           | 0.482846/0.292252              | 0.421776/0.229096            |
| Solutions Generated by IRR GA          |                           |                                  |                                |                                |                              |
| NAL                                    | 104.506                   | 0.7216                           | 0.27                           | 0.299691/0.179408              | 0.364494/0.241060            |
| NAL (Modified)                         | 99.811                    | 0.4281                           | 0.28                           | 0.395359/0.213128              | 0.346513/0.214314            |
| NAL (Ratio)                            | 90.665                    | 0.7772                           | 0.55                           | 0.435417/0.239017              | 0.374775/0.251001            |
| NAH                                    | 108.422                   | 0.558                            | 1.44                           | 0.362441/0.191724              | 0.373888/0.230802            |
| NAH (Modified)                         | 118.336                   | 0.216                            | 0.89                           | 0.247890/0.130361              | 0.324134/0.188822            |

## 6 CONCLUSIONS

A new evolutionary based representation combining redundancy and implicit fitness constraints was introduced to represent and search for synthesis design solutions in unstructured problem domains. The implicit redundant representation genetic algorithm (IRR GA) developed provided the encoding of a variable number of parameters, location independence of design variables, and the ability of the representation to self-organize through the use of redundancy. The IRR GA represented and searched for design synthesis alternatives in a highly unstructured problem domain. Two levels of unstructured frame problem formulations were examined to determine the effectiveness of the IRR GA representation on synthesizing frame design solutions. Constraining the allowable spacing of the x-coordinates along each floor, in addition to imposing member and nodal symmetry penalties, aided the synthesis process by reducing the number of possible combinations of nodal coordinates in the search space.

The IRR GA was able to obtain novel design solutions using an unstructured problem formulation that minimized volume while maximizing the floor space subject to satisfying stress, deflection, and symmetry penalties. The novel frame designs evolved by the IRR GA synthesis design method compare favorably with traditional frame design solutions obtained by trial and error. The results obtained for the IRR GA synthesis of frame design solutions reinforced the benefits of providing topology and geometry synthesis using an unstructured formulation and IRR GA without requiring the statement of heuristic rules to add or remove members or the definition of a ground structure topology and geometry for the design domain.

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