





# Ant Colony Optimization

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## 1 INTRODUCTION

Real Ants are capable of finding the shortest path from a food source to their nest by exploiting pheromone information. While walking, ants deposit pheromone trails on the ground and follow pheromone previously deposited by other ants. The above behavior of real ants has inspired the Ants System (AS), an algorithm in which a set of artificial ants cooperate to the solution of a problem by exchanging information via pheromone deposited on a graph. In the AS applied to the Traveling Salesman Problem (TSP), a set of cooperating agents, called ants, cooperate to find good solutions to TSP's through pheromone trails that they deposit on the edges of the TSP graph while building solutions. Informally, each ant constructs a TSP solution in an iterative way. Memory takes the form of pheromone trails deposited by ants on TSP edges. There are two reasons to use the AS on the TSP: i) The TSP graph represents the solution space of this problem. This TSP graph is used to describe the space where the ants walk (AS graph), ii) The AS transition function has goals similar to the TSP objective function. That is not the case for other combinatorial optimization problems. We propose a new distributed algorithm based on AS concepts, called the Combinatorial Ant System (CAS), to solve Combinatorial Optimization Problems (COP).

## 2 OUR APPROACH: THE COMBINATORIAL ANT SYSTEM

*a) Building the AS graph:* The first step is to build the COP graph, then we define the AS graph with the same structure of the COP graph. The AS graph has two weight matrices: the first one is defined according to the COP graph and registers the relationship between the elements of the solution space (COP matrix). The second one registers the pheromone trail accumulated on each edge (pheromone matrix). This weight matrix is calculated/updated according to the pheromone update formula. When the incoming edge weights of the pheromone matrix for a given node become higher, this node has similarly, a higher probability to be visited. If an edge between two nodes of the COP matrix is low, it

means that ideally if one of these nodes belongs to the final solution, the other one must belong too. If the edge is equal to infinite, it means that they are incompatible.

*b) Defining the transition function and the pheromone update formula:* The state transition rule depends on the ant traffic and the trail intensity at a given time. The trail intensity at a given time is defined by the pheromone update formula. The state transition rule and the pheromone update formula are built using the objective function of the COP. The transition function between nodes is given by:

$$Tf(\gamma_{rs}(t), Cf_{r \rightarrow s}^k(z)) = \gamma_{rs}(t) / Cf_{r \rightarrow s}^k(z)$$

Where  $Cf_{r \rightarrow s}^k(z)$  is the cost of the partial solution that is being built by the ant  $k$  when it crosses the edge  $(r, s)$  if it is in the position  $r$ , and  $z-l$  is the current length of the partial solution (current length of  $A^k$ ). The transition probability is calculated according to the equation:

$$P_{rs}^k(t) = \begin{cases} Tf(\gamma_{rs}(t), Cf_{r \rightarrow s}^k(z)) / \sum_{u \in J_r^k} Tf(\gamma_{ru}(t), Cf_{r \rightarrow u}^k(z)) & \text{If } s \in J_r^k \\ 0 & \text{Otherwise} \end{cases}$$

We use the classical pheromone updating rule, where the quantity per unit of length of trail substance laid on edge  $(r, s)$  by the  $k^{\text{th}}$  ant in that iteration ( $\Delta\gamma_{rs}^k(t)$ ) is calculated according to the following formula:

$$\Delta\gamma_{rs}^k(t) = \begin{cases} 1 / C_f^k(t) & \text{If edge } (r, s) \text{ has been crossed by ant } k \\ 0 & \text{Otherwise} \end{cases}$$

Where  $C_f^k(t)$  is the value of the cost function (objective function) of the solution proposed by ant  $k$  at iteration  $t$ . In this way, we can solve any COP. The COP graph defines the structure of the AS graph. Each ant builds a solution walking through this graph according to a transition rule and a pheromone update formula defined according to the objective function of the COP.

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## Are COMPETants more competent for problem solving? - the case of a routing and scheduling problem

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This paper presents a new approach based on the Ant Colony Optimization meta-heuristic as described in Bonabeau et al. (1999). In particular we propose a multi-colony algorithm to exploit the following characteristics.

A main ingredient of an ACO algorithm is a heuristic rule which guides the ants through the solution space. For most combinatorial optimization problems more than one heuristic algorithm exists. This is due to the complexity of these problems and the fact, that the heuristics are based on problem specific knowledge and usually exploit different problem characteristics. Thus, each heuristic solves the problem, but the solution quality usually depends on the actual problem constellation. Especially if the structure of the problem is not a priori obvious it is hard to choose the appropriate heuristic. One could either try a single heuristic and rely on its effectiveness for the problem at hand, or try a number of heuristics and choose the best result. Both approaches have their advantages and drawbacks. The former will generally be fast but may lead to solutions which are far from optimal, while the latter will generally achieve better results using however more computational power.

Given these difficulties in determining which heuristic rule to choose our approach can be described as follows. We utilize two colonies of ants, which solve the problem using different heuristic information. After the two colonies have been executed, ants choose which population they want to belong to in the next iteration. Thus, the population sizes of the colonies will change adaptively according to the solution quality associated with each colony, as the ants strive to belong to the more 'successful' population. Furthermore, when constructing their solutions the ants observe not only the pheromone information from their own population but also from the other population. Again they decide which information to utilize. While the first mechanism leads to an adaptation of the use of the

heuristic information, the latter mechanism focuses on the use of the adaptive memory. Thus, the first mechanism will lead to a reinforcement of the heuristic rule appropriate for the problem instance in general, while the second mechanism will guide ants to use the heuristic rule appropriate for different regions of the search space. Through these information flows, different patterns of good solutions are communicated between the populations and the overall solution quality should be improved.

A multi-colony approach has also been proposed in Gambardella et al. (1999). However, there the colonies solve different sub-problems with different objectives and communicate the best solutions. While this approach is based on the idea that different sub-problems call for different heuristics, our approach exploits the problem-solving capabilities of different heuristics for the same problem.

Preliminary results for a multiple objective transportation problem show that both information spillovers and an endogeneous determination of the appropriate heuristic rule improve solution quality. Using both mechanisms leads to the best results. Furthermore, our results show, that even if the appropriate heuristic rule were known in advance and embedded in a single colony ACO algorithm the results are not significantly better than our new approach. Thus, wasting computational resources for an inappropriate rule is outweighed by the advantages gained through information spillovers in our multi-colony approach.

### References

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- Gambardella, L. M., E. Taillard, G. Agazzi. 1999. MACSVRPTW: A Multiple Ant Colony System for Vehicle Routing Problems with Time Windows. D. Corne et al. (eds.) *New Ideas in Optimization*, Mc Graw-Hill, London.

# A new Ant Colony Algorithm for the Job Shop Scheduling Problem

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## SUMMARY

The dynamic evolution in many fields of the economy, caused by the increasing networking of the companies, supported by the Internet, and the shorter response times expected with that (e.g. on production requests) requires also in the production planning new and rapid, however also robust means (e.g. for ASP). The production planning has a direct influence onto the compliance of the dates and with that the customer contentment. The Job Shop Scheduling Problem (JSSP) is a section of production planning. In the recent Paper we present an Ant Colony Algorithm for the Job Shop Scheduling Problem (JSSP). The fun-

tance, and thus, an significant influence on the solution quality. Therefore it is not decisive which operation follows another operation, but rather the arrangement in the entire generated sequence, under observation of the technology restrictions. For this reason we employ a position-operation-pheromone-matrix (inspired by Merkle [2]) with the dimensions (*POS* x *ONR*), instead of an operation-operation-pheromone Matrix.

The following transition rules are used:

$$p_{pos,onr} = \frac{[\tau_{pos,onr}]^\alpha \cdot [\eta_{pos,onr}]^\beta}{\sum_{h \in \mathcal{PO}} [\tau_{pos,h}]^\alpha \cdot [\eta_{pos,h}]^\beta} \quad (1)$$

$$p_{pos,onr} = \frac{(\sum_{k=1}^{pos} [\gamma^{pos-k} \tau_{k,onr}])^\alpha \cdot [\eta_{pos,onr}]^\beta}{\sum_{h \in \mathcal{PO}} (\sum_{k=1}^{pos} [\gamma^{pos-k} \cdot \tau_{k,h}])^\alpha \cdot [\eta_{pos,h}]^\beta} \quad (2)$$

$$\eta_{pos,onr} = \frac{1}{p_{onr}}$$

| Job / Operation | onr | $\tau_{(7,8)}$      |
|-----------------|-----|---------------------|
| 2 2             | 8   | 0 0 0 0 0 1 ②       |
| 1 2             | 7   | 0 0 2 0 0 0 0       |
| 6 1             | 6   | 0 0 0 0 1 0 0       |
| 5 1             | 5   | 0 2 0 0 0 1 0       |
| 4 1             | 4   | 0 0 1 1 0 2 1       |
| 3 1             | 3   | 1 0 0 2 0 3 1       |
| 2 1             | 2   | 0 0 0 1 3 0 0       |
| 1 1             | 1   | 2 0 0 0 1 0 0       |
|                 |     | 1 2 3 4 5 6 7 ← pos |

Figure 1: position-operation-pheromone-matrix

amental idea of our work is the use of an Ant Colony Algorithm for the decision, which operation is to be scheduled next. The working method of the algorithm is comparable to the Ant System employed by Dorigo [1]. Every ant generates a feasible solution under use of a transition rule. In the calculation a heuristics value and a pheromone value, according to the current position, are considered[1]. At the JSSP the technology with the predecessor-successor relationships, the sequence, is more important. So the position of an operation in a solution (permutation), has a high impor-

Equation 1 is the originally calculation rule after Dorigo [1]. Equation 2 was implemented according to Merkle [2], called weighted summation evaluation.

The results show that the weighting of the pheromone amount depending on the distance to the current position supplies an improvement of the solution quality.

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## References

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